Various Aspects of Modality

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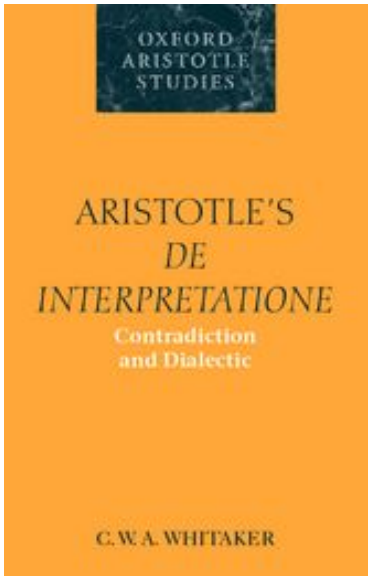
Alethic Logics

Alethic logic is the study of *necessity* and *possibility*.

$$\Box A \equiv A \text{ is necessarily true.}$$
$$\Diamond A \equiv A \text{ is possibly true.}$$

Alethic Logics

- Aristotle.
  On Interpretation.
  C. W. A. Whitaker, Aristotle's De interpretatione. Contradiction and Dialectic, Oxford: Clarendon Press, 1996.



Provability Logics

Provability logic is the study of *provability* in a formal system.

Gödel-Löb Provability Logic:

$$\Box A \equiv A \text{ is provable in Peano arithmetic.}$$
$$\Diamond A \equiv A \text{ is consistent with Peano arithmetic.}$$

Provability Logics

- Kurt Gödel.
  Eine Interpretation des intuitionistischen Aussagenkalkiils.
  Kurt Gödel, Collected Works, Vol. I, 300–303.
- M. H. Löb
  Solution of a problem of Leon Henkin.
  Journal of Symbolic Logic, 20 (1955), 115–118.
- R. Solovay.
  Provability interpretations of modal logic.
  Israel Journal of Mathematics, 25 (1976), 287-304.

Epistemic Logics I

Epistemic logic is the study of *knowledge* and *belief*.

Logic of knowledge:

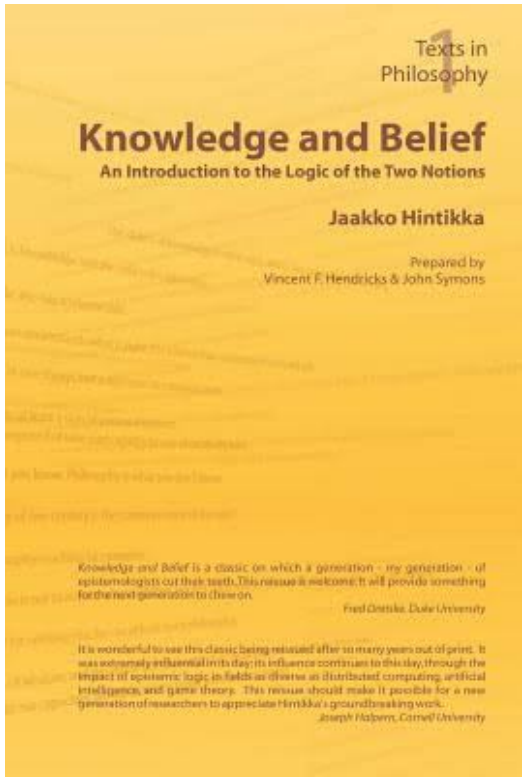
$$\begin{aligned} KA &\equiv A \text{ is known.} \\ PA &\equiv A \text{ is compatible with everything the agent knows.} \end{aligned}$$

Logic of belief:

$$\begin{aligned} BA &\equiv A \text{ is believed.} \\ CA &\equiv A \text{ is compatible with everything the agent believes.} \end{aligned}$$

Epistemic Logics I

J. Hintikka.
Knowledge and Belief.
Ithaca, N.Y., Cornell University Press (1962)



Epistemic Logics II

Group knowledge:

$K_i A \equiv$ Agent i knows that A.

$EA \equiv$ Every agent knows that A.

$DA \equiv$ A is *distributed knowledge* between agents, i.e.
A can be derived from the knowledge of all agents put together.

$CA \equiv$ A is *common knowledge* between agents, i.e.
$CA = EA \wedge EEA \wedge EEEA \wedge \ldots$

Epistemic Logics II

- R. Fagin, J. Y. Halpern, Y. Moses, M. Y. Vardi.
  Reasoning about Knowledge.
  MIT Press, 1995.




Justification Logics

Justification Logic is the study of *justifications*, *evidence* and *proofs*.

Epistemic interpretation:

$$t : A \equiv t \text{ is a justification for } A.$$

Provability interpretation:

$$t : A \equiv t \text{ is a proof of } A \text{ in Peano arithmetic.}$$

Justification Logics

- K. Gödel.
  Vortrag bei Zilsel.
  Kurt Gödel Collected Works, (S. Feferman et al., editor), Vol. III, pages 86–113.
- S. Artemov.
  Operational modal logic.
  Technical Report MSI 95–29, Cornell University, 1995.

Justification Logics

- S. Artemov, M. Fitting.
  Justification Logic.
  Stanford Encyclopedia of Philosophy,
  http://plato.stanford.edu/archives/fall2011/entries/logic-justification/
- Roman Kuznets's Homepage.
  Justification Logic Bibliography.
  http://www.iam.unibe.ch/ kuznets/JLBibliography.html
- Meghdad Ghari's Homepage.
  Justification Logic Bibliography in Persian.
  https://sites.google.com/site/meghdadghari/persianjlbibliograhpy

Temporal Logics

Temporal logic is the study of *time*.

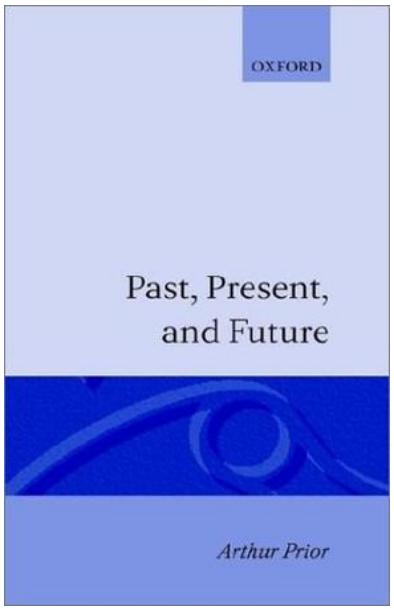
Prior's tense operators:

$GA \equiv$ At all times in the future A is true.
$FA \equiv$ At some time in the future A is true.

$HA \equiv$ At all times in the past A is true.
$PA \equiv$ At some time in the past A is true.

Temporal Logics

- Arthur N. Prior.
  Time and Modality.
  Clarendon Press, Oxford, 1957.
- Arthur N. Prior.
  Past, Present and Future.
  Clarendon Press, Oxford, 1967.



Dynamic Logics

Dynamic logic is the study of *programs* and *actions*.

$[\alpha]A \equiv$ It is necessary that after executing α, A is true.

$\langle\alpha\rangle A \equiv$ There is a computation of α that terminates in a state satisfying A.

$\alpha;\beta \equiv$ Execute α, then execute β.

$\alpha \cup \beta \equiv$ Choose either α or β nondeterministically and execute it.

$\alpha^* \equiv$ Execute α a nondeterministically chosen finite number of times.

$?A \equiv$ Test A; proceed if true, fail if false.

Dynamic Logics

V. Pratt.
Semantical considerations on Floyd-Hoare logic.
In Proceedings 17th IEEE Symposium on Computer Science, pages 109–121, 1976.

Deontic Logics

Deontic logic is the study of *obligation* and *permission*.

$$
\begin{aligned}
OA &\equiv \text{It is obligatory that } A.\\
PA &\equiv \text{It is permitted that } A.\\
FA &\equiv \text{It is forbidden that } A.
\end{aligned}
$$

Deontic Logics

E. Mally.
The Basic Laws of Ought: Elements of the Logic of Willing.
Universitäts-Buchhandlung Graz, 1926.

Conditional Logics

Conditional logic is the study of *conditionals*.

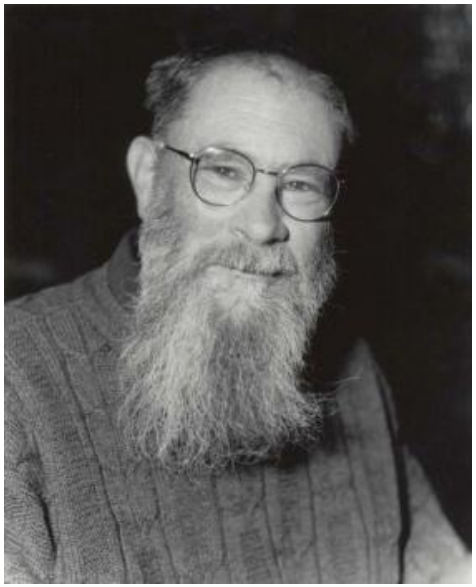
$[A]B \equiv$ if A and *ceteris paribus* (other things being equal), then B.

Conditional Logics

R. Stalnaker.
A theory of conditionals.
Studies in Logical Theory, American Philosophical Quarterly, Oxford, pages 98–112, 1968.

Conditional Logics

- D. Lewis.
  Counterfactuals.
  Oxford: Basil Blackwell Ltd, 1973.



- B. Chellas.
  Basic conditional logic.
  Journal of Philosophical Logic 4, 133–153, 1975.

Hybrid Logic

Hybrid Logic is the study of *nominals*.

$@_i A \quad \equiv \quad A$ is true in the unique state named by the nominal i.

Hybrid Logic


A. Prior.
Papers on Time and Tense.
Oxford: Clarendon Press, 1968.


R. Bull.
An Approach to Tense Logic.
Theoria, 36: 282–300, 1970.

Modal Logic of Forcing

Modal Logic of Forcing is the study of *forcing extensions*.

$\Box A \equiv$ A is necessarily true in all forcing extensions.

$\Diamond A \equiv$ A is forceably or possibly true in some forcing extension.

Modal Logic of Forcing

J. D. Hamkins.
A simple maximality principle.
Journal of Symbolic Logic, 68(2):527–550, 2003.