Teacher Training University
Faculty of Mathematical Sciences

PHD QUALIFY EXAM, 1386
Algebraic Geometry

All varieties are considered over an algebraically closed field k.

1. If X and Y are affine varieties, show that $X \times Y$ is also an affine variety, and, if X and Y are irreducible, then $X \times Y$ is irreducible.

2. Find the coordinate ring of $X \times Y$ in terms of the coordinate rings of X and Y. Conclude that if $P \in k[x_1, \cdots, x_n]$ and $Q \in k[y_1, \cdots, y_m]$ are prime ideals, then $P + Q$ is a prime ideal in $k[x_1, \cdots, x_n, y_1, \cdots, y_m]$.

3. Briefly, prove that if $X \subseteq \mathbb{P}^m$ is a quasi–projective variety, then X is birational to a hypersurface in $\mathbb{P}^m$ for some m.

4. If X is a quasi–projective variety and $x \in X$, then x has a neighborhood isomorphic to some affine variety.

5. Prove that any finite morphism of an affine variety is surjective.

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