HOMOLOGICAL DIMENSIONS; DEPTH FORMULA AND GRADE

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Let M and N be finite modules over a local ring (R, m). We say M and N satisfy the Depth formula provided:

$$\text{depth}_R(M \otimes_R N) = \text{depth}M + \text{depth}N - \text{depth}R$$

The following result is shown by M. Auslander:

Let (R, m) be a local ring; M and N finite R-modules with proj.dim $M < \infty$; $s = \sup\{i \in \mathbb{N}_0 | \text{Tor}_i^R(M, N) \neq 0\}$. If $\text{depth}_R\text{Tor}_s^R(M, N) \leq 1$ or $s = 0$, then

$$\text{depth}_R\text{Tor}_s^R(M, N) = \text{depth}_R M + \text{depth}_R N - \text{depth}R + s$$

Later C. Huneke and R. Wiegand showed that the finite Tor-independence modules over complete intersection rings satisfy depth formula. Also S. Iyengar proved that the finite Tor-independence modules satisfy depth formula provided one of them has finite complete intersection dimension. In this talk we give some generalizations by replacing modules with complexes. Furthermore we use the notion of grade and some homological dimensions, and we give some inequalities.