Bush-type Hadamard matrices; existence and applications

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Abstract

Bush–type Hadamard matrices have been around for over thirty years now. However, only recently some great potential for the use of these matrices were discovered. In this survey talk we will see that a

- Bush–type Hadamard matrix of order $4n^2$, where $q = (2n-1)^2$ is a prime power, can be used to generate twin symmetric designs with parameters $\big(4n^2(q^m + q^{m-1} + \cdots + q + 1), (2n^2 - n)q^m, (n^2 - n)q^m\big)$, for each positive integer m,

- Bush–type Hadamard matrix of order $4n^2$, where $q = (2n+1)^2$ is a prime power, can be used to generate Siamese twin symmetric designs with parameters $\big(4n^2(q^m + q^{m-1} + \cdots + q + 1), (2n^2 + n)q^m, (n^2 + n)q^m\big)$, for each positive integer m,

- block negacyclic Bush–type Hadamard matrix of order $4n^2$, n odd, where $q = (2n-1)^2$ is a prime power, can be used to generate twin Strongly Regular Graphs with parameters $\big(4n^2(q^m + q^{m-1} + \cdots + q + 1), (2n^2 - n)q^m, (n^2 - n)q^m, (n^2 - n)q^m\big)$, for each positive integer m,

- block negacyclic Bush–type Hadamard matrix of order $4n^2$, n odd, where $q = (2n+1)^2$ is a prime power, can be used to generate Siamese twin Strongly Regular Graphs with parameters $\big(4n^2(q^m + q^{m-1} + \cdots + q + 1), (2n^2 + n)q^m, (n^2 + n)q^m, (n^2 + n)q^m\big)$, for each positive integer m.

The existence (and for some orders the abundance) of Bush–type Hadamard matrices will be discussed. The most surprising part of recent findings is the existence of a symmetric Bush–type Hadamard matrix of order 324.