Results and Problems on Grundy and First-Fit Coloring of Graphs

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Given a graph G, by a *Grundy k-coloring* of G we mean any k-vertex coloring of G such that for each two colors i and j, $i < j$, every vertex of G colored by j has a neighbor with color i. The maximum k for which there exists a Grundy k-coloring is denoted by $\Gamma(G)$ and called *Grundy (chromatic) number* of G.
In this talk we first review previous related works and then report the results of the author. We show that for fixed k, deciding whether $\Gamma(G) \geq k$ can be made in polynomial time. But in general, Grundy number is an NP-Complete problem. We show that it is an NP-Complete problem even for the complement of bipartite graphs. We propose an additive Nordhaus-Gaddum type inequality concerning $\Gamma(G)$ and $\Gamma(G^c)$, and prove it in some cases like trees, split graphs and a family of bipartite graphs and conjecture its truth in general. We introduce *well-colored* graphs, which are graphs G for which applying every greedy coloring results in a minimum coloring of G. Equivalently G is well-colored if $\Gamma(G) = \chi(G)$. Recognition problem of well-colored graphs is a coNP problem. But we show that recognizing well-colored, the complement of bipartite graphs is a problem with a polynomial time solution. We conjecture that this problem belongs to the class P. The greedy procedure as an on-line algorithm is called *First Fit coloring* and denoted by FF. The worst case behavior of FF on a graph G is in fact the same as $\Gamma(G)$. Using *atoms*, we determine the exact behavior of FF on the class of graphs with girth $g > 4$. Precisely, if $\mathsf{FF}(n) = max\{\mathsf{FF}(G) : g(G) > 4, |V(G)| = n\}$, then $\mathsf{FF}(n) = \Theta(\sqrt{n})$. At last, we show a relationship between Grundy coloring of graphs and $L(1,1)$-labeling of graphs. This later concept comes from radio frequency assignment of graphs in networking and it is a coloring of a graph G, in which any two vertices with distance at most 2 take different colors. We show that these concepts coincide with each other for some atoms and it is when we have $\Gamma(G) = \chi_{1,1}(G)$. We use this fact in our constructions.