

Automorphisms of Models of Arithmetic and Set Theory

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Inspired by the Kunen method of iterated ultrapowers in the theory of large cardinals, we devise a new method of building automorphisms of countable recursively saturated models of Peano arithmetic which subsumes results of Smorynski, Kaye, Kossak, Kotlarski, and Schmerl. This new method not only helps to strengthen and unify the known results in the case of arithmetic, but also leads to new results regarding automorphisms of models of set theory. For example, we introduce the set theoretical analogue of the Paris-Kirby arithmetical notion of strong cuts to prove the following result.

Theorem. Suppose M is a countable recursively saturated model of ZFC, and suppose I is a proper initial segment of the ordinals of M . The following are equivalent:

- (1) I is a strong cut of M .
- (2) There is an embedding e of the automorphism group $\text{Aut}(\mathbb{Q})$ of the ordered set of rationals into the automorphism group $\text{Aut}(M)$ of M , such that whenever f in $\text{Aut}(\mathbb{Q})$ has no fixed points, then $e(f)$ is an automorphism of M whose fixed point set is precisely I .