Additive Polynomials and Their Role in the Model Theory of Power Series Fields over Finite Fields and in Local Uniformization

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A polynomial f over an infinite field K is called additive if $f(a+b) = f(a) + f(b)$ for all $a, b \in K$. If the characteristic of K is 0, then the only additive polynomials are of the form cx with $c \in K$. But if the characteristic is $p > 0$, then for instance, X^p and the Artin-Schreier-polynomial $X^p - X$ are additive. I will explain the particular role that additive polynomials play in the model theory of power series fields over finite fields. This is tightly connected with the structure theory of valued function fields, which in turn also plays a crucial role for the problem of local uniformization. The latter is a local form of resolution of singularities and therefore of a valuation theoretical nature. While local uniformization has been proved in characteristic 0 by Zariski in 1940, the positive characteristic case is still open, and only special cases have been solved. In all of these solutions, additive polynomials play a role. Finally, I will sketch some results on the classification of certain Artin-Schreier-extensions of valued fields and their connection with recent work of Cutkosky and Piltant on resolution of singularities in positive characteristic.