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Optimization Problems for the Eigenvalues of Laplace and Schrödinger Operators

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Problems linking the geometry of a domain to the eigenvalues of its (Dirichlet or Neumann) Laplacian are studied intensively since the years 1920's where Faber and Krahn solved independently the Rayleigh conjecture: The round ball minimizes the first Dirichlet eigenvalue among all the domains of given volume.

In the case where the domain is a closed manifold, the geometry is given by a Riemannian metric. The first result in this setting was obtained by Hersch in 1970: On the 2-sphere, the standard metric maximizes the first positive eigenvalue among all the metrics of given volume.

Our aim is to present some results extending the classical ones to higher eigenvalues, other manifolds, and to Schrödinger type operators. Also, we introduce the notion of critical domain and of critical metric for the eigenvalues and give some necessary and sufficient conditions for a domain, or a Riemannian metric, to be critical.