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Combinatorial Results Using Inner Products

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In this paper we present some combinatorial applications of the inner product of vectors. We prove that if p is a prime number and n, c are two integers and and $\mathcal{A}, B$ are two collections of subsets of $\{1, ..., n\}$ such that for any $A \in \mathcal{A}$, and for any $B \in \mathcal{B}$, $|A \cap B| \equiv c$ (mod p), then for $c \neq 0$, $|\mathcal{A}||B| \leq 2^{n-1}$, and for $c = 0, |\mathcal{A}||B| \leq 2^n$. In 1985 Hadamard graphs were defined by Ito Noboru. An Hadamard Graph $\Delta(n)$ is a graph whose vertices are all -1,1-vectors of length n and two vertices are adjacent if their inner product is zero. We note that there is an Hadamard matrix of order n if and only if the clique number of $\Delta(n)$ is n. In this paper we introduce the negative Hadamard graphs. Let $V_n = \{\pm 1\}^n$. We construct a graph Γ_n with vertex set V_n in which two vertices u and v are adjacent if $u.v < 0$. We call this graph the *negative Hadamard graph* of order $n + 1$. We prove that this graph is vertex transitive and determine the domination number, the edge chromatic number and the structure of the automorphism group of this graph. In particular we prove that for $n \geq 4$ and $n \equiv 2$ or 3 (mod 4), the automorphism group of Γ_n is isomorphic to $S_n \times \mathbb{Z}_2^n$.