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Rings of Quotients of $C(X)$

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Let X be a topological space and $T'(X)$ be the ring of all real-valued functions on X by pointwise addition and multiplication such that for every $f \in T'(X)$ there exists a dense subset D in X which is an open subset and $f \mid D$ is continuous. A commutative ring R is said to be (von Neumann) regular if, for each $r \in R$, there exists an $s \in R$ such that $r^2 s = r$. the element rs is idempotent, hence so is $1 - rs$. Since $r(1 - rs) = 0$, each element of R is either a zero-divisor or a unit. In this talk we give the following results.
Theorem1.1 Let X be a topological space. Then $T'(X)$ is a regular commutative ring.
Theorem1.2 Let X be a topological space. The following conditions are equivalent.

1) $T'(X)$ is a ring of quotients of C(X), the ring of continuous real-valued functions on X.

2) The maximal ring of quotients of $C(X)$ is $T'(X)$.

3) X is a discrete space.

Theorem1.3 Let X be a Tychonoff space and let $Q(X)$ be the maximal ring of quotients of $C(X)$. Then X is an almost discrete space if and only if the socle of $Q(X)$ is an essential ideal in $Q(X)$.
We give topological characterizations of minimal ideal and socle of $T'(X)$ and get new results.

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Asymptotic Primes of Ratliff-Rush Closure of Ideals with Respect to Modules

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This is a joint work with R. Naghipour.

Let R be a commutative Noetherian ring, M a non-zero finitely generated R-module and I an ideal of R. The purpose of this paper is to develop the concept of Ratliff- Rush closure $\widetilde{I}^{(M)}$ of I with respect to M. It is shown that the sequence $Ass_R R/\widetilde{I^n}^{(M)},\ n = 1, 2, ...,$ of associated prime ideals is increasing and eventually stabilizes. This result extends Mirbagheri-Ratliffs main result in *On the relevant transform and the relevant component of an ideal*, J. Algebra 111 (1987), 507-519. Furthermore, if R is local, then the operation $I \to \widetilde{I}^{(M)}$ is a c^*-operation on the set of ideals I of R, each ideal I has a minimal Ratliff-Rush reduction J with respect to M, and, if K is an ideal between J and I, then every minimal generating set for J extends to a minimal generating set of K.

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$Max_R(M)$ and Zariski Topology

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Let R be a commutative ring and let M be an R-module. Let $X = Spec_R(M)$ be the prime spectrum of M with Zariski topology. In this paper, by using the topological properties of X, we will obtain some conditions under which $Max_R(M) = Spec_R(M)$.

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Tilting Theory and its Applications

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This is a joint work with Sh. Salarian.

Tilting and cotilting theory for finite dimensional algebras started in early 1980s as a generalization of Morita theory of equivalence and duality for modules by Brenner and Butler [**BB**] and by Happel and Ringel [**HR**]. For generalizations of the notion to arbitrary rings see [**CF, Cp, Mi**]. Moritas Theorem states that for rings R and S the corresponding module categories are equivalent as abelian categories if there exists a progenerator P for the categories of R-modules such that $S \cong End_R(P)^{op}$. Tilting theory help us to compare module categories which are somehow similar without necessarily being Morita equivalent. More explicitly, for a tilting module (more generally a tilting complex) T over a finite dimensional algebra A the categories of modules mod-A and mod-B, where B is the endomorphism ring of T, offer a lot of similarities despite the fact that mod-A and mod-B usually are far from being equivalent. The theory originates in the representation theory of finite dimensional algebras. Today it plays an important role in various branches of modern algebra, ranging from Lie theory and algebraic geometry to homological algebra. In this talk, we present the basic concepts of tilting theory as well as the variety of applications. We will also give some recent results, e.g. all tilting modules are of finite type, and that all cotilting modules are pure-injective. Finally, we will discuss their constructions over formal triangular matrix rings.

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Graded Local Cohomology: Attached and Associated Primes, Asymptotic Behaviors

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Assume that $R = \oplus_{i \in \mathbb{N}_0} R_i$ is a homogeneous graded Noetherian ring, and that M is a $\mathbb{Z}$-graded R-module, where $\mathbb{N}_0$ (resp. $\mathbb{Z}$) denote the set all non-negative integers (resp. integers). The set of all homogeneous attached prime ideals of the top non-vanishing local cohomology module of a finitely generated module M, $H^c_{R_+}(M)$, with respect to the irrelevant ideal $R_+ := \oplus_{i \geq 1} R_i$ and the set of associated primes of $H^i_{R_+}(M)$ are studied. The asymptotic behavior of $\mathrm{Hom}_R(R/R_+, H^s_{R_+}(M))$ for $s \geq f(M)$ is discussed, where $f(M)$ is the finiteness dimension of M. It is shown that $H^h_{R_+}(M)$ is tame if $H^i_{R_+}(M)$ are Artinian for all $i > h$.

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A Survey on the Well-Behaviour of Injectivity in a Category

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This is a joint work with H. Haddadi.

Injectivity has always been of interest to mathematicians in almost every field of study. In classical mathematics, one usually defines injectivity with respect to monomorphism. In this talk we take an arbitrary category $\mathcal{A}$ and an arbitrary subclass $\mathcal{M}$ of the class of monomorphisms in $\mathcal{A}$ and study injectivity with respect to $\mathcal{M}$ and its behaviour with respect to other related notions such as retractness and essentialness. We will study some sufficient conditions for the so called well-behaviour of injectivity.

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Gorenstein Injective Modules and Auslander Categories

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This is a joint work with M. Tousi.

In this paper, we study Gorenstein injective modules over a local Noetherian ring R. For an R-module M, we show that M is Gorenstein injective if and only if $\mathrm{Hom}_R(\hat{R}, M)$ belongs to Auslander category $B(\hat{R})$, M is cotorsion and $\mathrm{Ext}^i_R(E, M) = 0$ for all injective R-modules E and all $i > 0$.

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Strict Complete Intersection versus Set Theoretic Complete Intersection

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Let Y be a d-dimensional projective variety in $\mathbb{P}^n$ and $\mathfrak{a}$ its definition ideal. Let $f_1, \cdots, f_s$ be a minimal generating set for $\mathfrak{a}$. Then $Y = V(\mathfrak{a}) = V((f_1, \cdots, f_s))$.

Definition. The variety Y is called *strict(ideal theoretic) complete intersection* if $s = n - d$, and is called *set theoretic complete intersection* if it can be represented as the intersection of $s = n - d$ hypersurfaces. In other words there are $n - d = s$ hypersurfaces $H = V(g_1), \cdots, H_s = V(f_s)$ such that

$$Y = H_1 \cap \cdots \cap H_s = V(g_1) \cap \cdots, \cap V(g_s) = V(\sqrt{(g_1, \cdots, g_s)}).$$

It is trivial that if Y is strict complete intersection then it is set theoretic complete intersection, but the converse is not true. The twisted cubic curve in $\mathbb{P}^3$is an example of such a curve which is set theoretic complete intersection but its ideal of definition can not be produced by two elements. Generally, determining whether a variety is a set theoretic complete intersection is a difficult task. Even it is still an unsolved problem that every curve in $\mathbb{P}^3$ is a set theoretic complete intersection.

In this talk we consider algebraic counterpart of this problem and state special cases which one can determine an ideal I is a set theoretic complete intersection and compare the properties which these two conditions imply.

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On the Cofiniteness Properties of Local Cohomology Modules

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Let I be an ideal of a commutative Noetherian ring R and let M be a finitely generated R-module. For a fixed n, we study the finiteness properties of $Ext^i_R(R/I, H^n_I(M))$ in several cases for all $i \geqslant 0$.

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Finiteness and Cofiniteness of Local Cohomology Modules

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This is a joint work with K. Khashyarmanesh.

Let R be a commutative Noetherian ring, I an ideal of R, and M an R-module. The notion of Local cohomology was first introduced and studied by Grothendieck in 1968. It is well known that the local cohomology modules $H_I^n(M)$ are not generally finitely generated for $n \geqslant 0$. So the natural questions concerning the local cohomology theory are the following questions:

(1) When is $H_I^n(M)$ finitely generated?

(2) What annihilates $H_I^n(M)$?

(3) When is $H_I^n(M)$ I-cofinite?

In this talk we are going to study these questions and to find some relations between them.

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Finiteness Results of Local Cohomology Modules

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This is a joint work with H. Saremi.

Let R be a commutative Noetherian local ring, $\mathfrak{a}$ an ideal of R, and M a finitely generated generalized f-module. Let t be a positive integer such that $H^t_{\mathfrak{a}}(M) \neq 0$ and $t > \mathrm{d}imM - \mathrm{d}imM/\mathfrak{a}M$. Then there exists an ideal $\mathfrak{b} \supseteq \mathfrak{a}$ such that

(1) $\mathrm{d}imM - \mathrm{d}imM/\mathfrak{b}M$
(2) the natural homomorphism $H^i_{\mathfrak{b}}(M) \to H^i_{\mathfrak{a}}(M)$ is an isomorphism for all $i > t$ and is surjective for $i = t$.

Also, we show that if $Supp_R(H^i_{\mathfrak{a}}(M))$ is a finite set for all $i < t$, then there exists an ideal $\mathfrak{b}$ of R such that $\dim R/\mathfrak{b} \leq 1$ and $H^i_{\mathfrak{b}}(M) \cong H^i_{\mathfrak{a}}(M)$ for all $i < t$.

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Cover and Projective Cover of Cyclic Acts over a Monoid

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In [1] Bican, Bashir and Enochs finally solved a long standing conjecture in module theory that all modules over a unitary ring have a flat cover. The only substantial work on covers of acts over monoids seems to be that of Isbell, Fountain and Kilp ([3], [2], [4]) who only consider projective covers. In contrast to the case of modules over a ring, the limit preservation properties of tensor product functor for acts over a monoid gives different kinds of flatness. The situation for flat covers of acts has not been addressed and [5] is an attempt to initiate such a study. Here, we consider almost exclusively covers of cyclic acts and restrict our attention to projective covers. We give a new necessary and sufficient condition for a cyclic act to have a projective cover and provide a new proof of one of Isbells classic results concerning projective covers. We also show that covers are not generally unique, unlike the situation for projective covers.

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Graphs from Rings

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We define some graphs associated to a commutative ring based on some properties of elements of R such as edge ideal, intersection graph, zero divisor graph, co-maximal graph, and etc. Three major problems in this area would at this time be: characterize the resulting graphs; when do two rings have isomorphic graphs; relate the structure of the graph to that of the rings.

In this talk we state a few results on these problems for each graphs.

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Δ-reductions and Δ-closures of Ideals with Respect to an Artinian Module

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The important ideas of reduction and integral closure of an ideal in a commutative Noetherian ring R (with non-zero identity) were introduced by Northcott and Rees [5]. We refer the reader to [2, Ch. 18] for the basic properties of reductions and integral closures of ideals. These concepts have been extended to ideals in an arbitrary commutative ring by L.J. Ratliff. Namely, the important notions of Δ-closure and Δ-reduction of an ideal in a commutative ring R were introduced and studied in [6] and [7] as a refinement of the reduction and integral closure of an ideal, and these new ideas have been proved useful in several questions, for example see [3], [4], [8].

Let R be a commutative ring (with non-zero identity) and let I be an ideal of R. Let Δ be an arbitrary multiplicatively closed set of non-zero ideals of R. The Δ-*closure of* I, is denoted by I_Δ, is defined to be the ideal

$$I_\Delta := \bigcup_{K\in\Delta}(IK :_R K) = \sum_{K\in\Delta}(IK :_R K)$$

of R. We say that I is Δ-*closed* if $I = I_\Delta$. Also, we say that I is a Δ-*reduction of* J in case $I \subseteq J \subseteq I_\Delta$.

The purpose of the present paper is to introduce dual concepts of Δ-reduction and Δ-closure of the ideal I with respect to an Artinian module A over a commutative ring R. We show that many of the general properties of standard Δ-reductions and Δ-closures extend to Δ-reductions and Δ-closures of ideals with respect to an Artinian module. Specifically, for an arbitrary multiplicatively closed set Δ of non-zero ideals of R and an Artinian module A over R, we define the Δ-closure $I_\Delta^{(A)}$ of an ideal I of R with respect to A and then we show that the operation $I \longrightarrow I_\Delta^{(A)}$ is a semiprime operation that satisfies the Δ-cancelation law: if $(IK)_\Delta^{(A)} \subseteq (JK)_\Delta^{(A)}$ and $K \in \Delta$, then $I_\Delta^{(A)} \subseteq J_\Delta^{(A)}$. Also, we show that whenever every element in Δ is finitely generated, then $I_\Delta^{(A)}$ is decomposable and each associated prime of $I_\Delta^{(A)}$ is an associated prime of $I_\Delta^{(A)}K$ and $(IK)_\Delta^{(A)}$ for all $K \in \Delta$. Finally, we will obtain a finiteness result about the asymptotic prime divisors, namely it is shown that if R is a complete local (Noetherian) ring and every element of Δ is A-coregular, then the sequence $\{\mathrm{Ass}_R R/(I^n)^{(A)}_{\Delta}\}_{n\geq 1}$ of associated prime ideals is increasing and eventually constant.

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On the Use of Sector Partitions

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This is a joint work with M. Tousi [ST].

In [MS, Remark 13.15], the authors posed the problem of which faces of a saturated affine semigroup Q correspond to prime ideals associated to the local cohomology module $H_I^i(\omega_R)$ where ω_R is the canonical module of the semigroup ring $R = \mathbb{k}[Q]$, $\mathbb{k}$ a field, and I is a monomial ideal in R. In this talk, we will give a solution in the case that Q is simplicial.

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On the Weil and Cartier Divisors

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This is a joint work with J. Asadollahi and F. Jahanshahi.

A Weil divisor on a normal variety X is a finite formal sum $D = \Sigma a_i D_i$, where the D_is are distinct irreducible hypersurfaces of X and $a_i \in \mathbb{Z}$. The set of all Weil divisors is a group under addition and is denoted by $D(X)$. The study of the Weil divisor group and its subgroup, Cartier divisor group, is an interesting subject in algebraic geometry and commutative algebra. In this talk, we introduce and study a subgroup of $D(X)$, which will be called Gorenstein divisor group. We show that this subgroup share several nice properties with the Cartier divisor subgroup. A good source for background and details of the theory is [B], see also [H].

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Duality for a Local Cohen-Macaulay Ring

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This is joint work with M.A. Esmkhani. Let $(R, \mathfrak{m})$ be a local Cohen-Macaulay ring. If R has a canonical module, then there are some interesting results about duality for this situation. The purpose of this talk is to show that indeed one may obtain similar results in the case where R does not have a canonical module.

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A Conjecture of Stanley

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This is a joint work with J. Herzog and A. Soleyman Jahan.

In this talk we discuss a conjecture of Stanley concerning a combinatorial upper bound for the depth of a $\mathbb{Z}^n$-graded module. Here we consider his conjecture only for S/I where I is a monomial ideal of the polynomial ring $S = K[x_1, \ldots, x_n]$ in n variables over a field K.

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On Isomorphism of Simplicial Complexes and Their Related Rings

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In this talk, we provide a simple proof for the fact that two simplicial complexes are isomorphic if and only if their associated Stanley-Reisner rings, or their associated facet rings are isomorphic as K-algebras. As a consequence, we show that two graphs are isomorphic if and only if their associated edge rings are isomorphic as K-algebras. Based on an explicit K-algebra isomorphism of two Stanley-Reisner rings, or facet rings or edge rings, we present a fast algorithm to find explicitly the isomorphism of the associated simplicial complexes, or graphs.

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Orderings in Commutative Rings, Nullstellensatz, and Positivstellensatz

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In this talk we first state Hilbert's 17 Problem and say a word about the fields on which we can define an order. Such fields have a real closure. The real closure of an ordered field is defined and then we introduce order on commutative rings. Similar to Hilbert Nullstellensatz in classical commutative algebra, there is a version in real algebra as real Nullstellensatz for real rings(i.e., the rings on which we can define an ordering). Finally, we state this real Nullstellensatz and Positivstellensatz. The talk is mainly based on [1,2,3].

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