5TH SEMINAR ON COMMUTATIVE ALGEBRA AND RELATED TOPICS

G-Gorenstein Modules

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Let R be a commutative Noetherian ring. In this paper, we study those finitely generated R-modules whose Cousin complexes provide Gorenstein injective resolutions. We call such a module a G-Gorenstein module. Characterizations of G-Gorenstein modules are given and a class of such modules is determined. It is shown that the class of G-Gorenstein modules strictly contains the class of Gorenstein modules. Also, we provide a Gorenstein injective resolution for a balanced big Cohen-Macaulay R-module. Finally, using the notion of a G-Gorenstein module, we obtain characterizations of Gorenstein and regular local rings.

Local Cohomology and Serre Suncategories

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The membership of the local cohomology modules $\mathrm{H}^n_{\mathfrak{a}}(M)$ of a module M in certain Serre subcategories of the category of modules is studied from below ($i < n$) and from above ($i > n$). Generalizations of depth and regular sequences are defined. The relation of these notions to local cohomology are found. It is shown that the membership

of the local cohomology modules of a finite module in a Serre subcategory in the upper range just depends on the support of the module.

Some Results On Prime And Semi-Prime Submodules

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Let R be a commutative ring and M be a unitary R-module. Our aim is to investigate semiprime submodule of a module and to consider some of its properties. The talk is based on a joint work with Rezvan Varmazyar.

Keywords: prime submodule, semiprime submodule, multiplication module, radical formula. 2000 Mathematics Subject Classification: 13C13, 13C99.

On the Min-Projective Modules

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In this paper, we first give some results of min-projective and min-flat modules. Then, we study the class of min-projective on some rings such as cotorsion, von Neumann regular and perfect rings.

Keywords: Min-projective modules, Min-flat modules, Von Neumann regular ring, perfect ring.
Mathematics Subject Classifications: Primary 16 D50; Secondary 16 E50.

On the Compactly Generated Homotopy Categories

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Let R be an associative ring and X be a class of R-modules which is closed with respect to arbitrary direct sums and direct summands. The homotopy category $\mathbb{K}(X)$ has been the subject of several recent researches, specially those done by S. Iyengar, P. Jørgensen, H. Krause and A. Neeman. These categories are introduced by Grothendieck in his studies of derived categories. Then Neeman studied those homotopy categories that are compactly generated, i.e. those that have a generating set consisting of compact objects. Roughly speaking, compact objects are like finitely generated modules over a ring. There are several formal results concerning compactly generated homotopy categories. In particular, we can discuss the existence of some special adjoints on these categories. So it is very important to decide whether a homotopy category has a generating set of compact object. In this talk, after brief reviewing the existing results we will study some compactly generated homotopy categories. The talk is based on a joint work with D. Murfet and Sh. Salarian.

On Gorenstein Cohomological Dimension of Groups

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Motivated by the notion of Auslander's G-dimension, we assign a numerical invariant to any group Γ. It provides a refinement of the cohomological dimension and fits well into the well-known hierarchy of dimensions assigned already to Γ. We study this dimension and show its power in reflecting the properties of the underlying group. Its connections to the relative and the Tate cohomology of groups also will be discussed. The talk is based on a joint work with J. Asadollahi and Sh. Salarian.

The Annihilating-Ideal Graph of Commutative Rings

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Let R be a commutative ring with $\mathbb{A}(R)$ its set of ideals with nonzero annihilator. We introduce and investigate the *annihilating-ideal graph* of R, denoted by $\mathbb{AG}(R)$. It is the (undirected) graph with vertices $\mathbb{A}(R)^* := \mathbb{A}(R) \setminus \{(0)\}$, and two distinct vertices I and J are adjacent if and only if $IJ = (0)$. First, we study some finiteness conditions of $\mathbb{AG}(R)$. For instance, it is shown that if R is not a domain, then $\mathbb{AG}(R)$ has ACC (resp., DCC) on vertices if and only if R is Noetherian (resp., Artinian). Moreover, the set of vertices of $\mathbb{AG}(R)$ and the set of nonzero proper ideals of R have the same cardinality when R is either an Artinian or a decomposable ring. This yields for a ring R, $\mathbb{AG}(R)$ has n vertices ($n \geq 1$) if and only if R has only n nonzero proper ideals. Next, we study the connectivity of $\mathbb{AG}(R)$. Also, rings R for which the graph $\mathbb{AG}(R)$ is complete or star, are characterized, as well as rings R for which every vertex of $\mathbb{AG}(R)$ is a prime (or maximal) ideal. Finally, we study the diameter and coloring of annihilating-ideal graphs.
Key Words: Commutative rings; Annihilating-ideal; Zero-divisor; Graph

2000 Mathematics Subject Classification: 13A15; 05C75.

Artinian and non-Artinian Local Cohomology Modules

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Let M be a finite module over a commutative noetherian ring R. For ideals $\mathfrak{a}$ and $\mathfrak{b}$ of R, the relations between cohomological dimensions of M with respect to $\mathfrak{a}, \mathfrak{b}$, $\mathfrak{a} \cap \mathfrak{b}$ and $\mathfrak{a} + \mathfrak{b}$ are studied. When R

is local, it is shown that M is generalized Cohen-Macaulay if there exists an ideal $\mathfrak{a}$ such that all local cohomology modules of M with respect to $\mathfrak{a}$ have finite lengths. Also, when r is an integer such that $0 \leq r < \dim_R(M)$, any maximal element $\mathfrak{q}$ of the non-empty set of ideals $\{\mathfrak{a} : \mathrm{H}^i_{\mathfrak{a}}(M)$ is not artinian for some $i, i \geq r\}$ is a prime ideal and that all Bass numbers of $\mathrm{H}^i_{\mathfrak{q}}(M)$ are finite for all $i \geq r$.

Injectivity and Essentiality

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Essentiality is an important notion closely related to **injectivity**. Depending on a class $\mathcal{M}$ of morphisms of a category $\mathcal{A}$, three different types of essentiality are considered in literature. Each has its own benefits in regards with the behaviour of $\mathcal{M}$-injectivity. In this paper we intend to study these different notions of essentiality and to investigate their relations to injectivity and among themselves. We will see, among other things, that although these essential extensions are not necessarily equivalent, they behave almost equivalently with regard to injectivity.

On the Homotopy Category of Representations of Quivers

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In this talk, we study compactness and total acyclicity for complexes of representations for some classes of quivers. We give a characterization for such objects in terms of the complexes obtained in each vertex. This is done, among other classes of quivers, for left rooted quivers. These include finite quivers and also those satisfying the property $*$, in the sense of the Enochs's school. The tree quivers and infinite linear quivers A^+_∞ and A^-_∞ also will be discussed. The talk is

based on a joint work with J. Asadollahi, R. Hafezi and Sh. Salarian.

Subacts of Multiplication S-acts

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Let S be a commutative monoid with zero and an S-act be a centered. This paper is devoted to the study of greatest common divisor and least common multiple of two subact in a multiplication S-act.Also we show that if $I, J \in T(S)$ and $\mathrm{lcm}(I, J)$ exists, then so too does $\gcd(I, J)$ and in particular $IJ = \gcd(I, J)\mathrm{lcm}(I, J)$, where $T(S)$ is the set strongly faithful multiplication ideals of S.

Adjoint Situations Between Zariski and Radical Functors

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Throughout all rings are commutative rings with identity and all modules are unitary. RAD is a full subcategory of the category R-Lattices $R-LAT$, whose objects are the lattices $RAD(M)$ depending on R-modules M and containing the radical submodules of M. Moreover it can be considered as a full subcategory of the category $ID(R)$-semimodules $ID(R)-SEM$, where $ID(R)$ is the semiring of ideals of R. Also ZAR is an another full subcategory of $ID(R)-SEM$, whose objects are the Zariski spaces $\zeta(M)$. Using a natural isomorphism between two functors $\mathcal{R}$ and $\mathcal{Z}$, we give two adjoint functors between RAD and ZAR. They help us to obtain some algebraic conclusions via categorical preserving objects and morphisms.

On the Endomorphism Rings of Local Cohomology Modules

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Let R be a commutative Noetherian ring and $\mathfrak{a}$ be a proper ideal of R. We show that if $n := \operatorname{grade}_R \mathfrak{a}$, then $\operatorname{End}_R(R) \cong \operatorname{Ext}^n_R(\operatorname{H}^n_{\mathfrak{a}}(R), R)$. We also prove that, for a non-negative integer n such that $H^i_{\mathfrak{a}}(R) = 0$ for every $i \neq n$, if $\operatorname{Ext}^i_R(R_z, R) = 0$ for all $i > 0$ and $z \in \mathfrak{a}$, then $\operatorname{End}_R(\operatorname{H}^n_{\mathfrak{a}}(R))$ is a homomorphic image of R, where R_z is the ring of fractions of R with respect to multiplicatively closed subset $\{z^j | j \geq 0\}$ of R. Also, if moreover $Hom_R(R_z, R) = 0$ for all $z \in \mathfrak{a}$, then $\mu_{\operatorname{H}^n_{\mathfrak{a}}(R)}$ is isomorphism, where $\mu_{\operatorname{H}^n_{\mathfrak{a}}(R)}$ is the canonical ring homomorphism $R \longrightarrow \operatorname{End}_R(\operatorname{H}^n_{\mathfrak{a}}(R))$.

Asymptotic Behaviour of Certain Sets of Associated Prime Ideals of Ext-Modules

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Let R be a commutative Noetherian ring, $\mathfrak{a}$ be an ideal of R and M be a finitely generated R-module. Melkersson and Schenzel asked whether the set

$$\operatorname{Ass}_R \operatorname{Ext}^i_R(R/\mathfrak{a}^j, M)$$

becomes stable for a fixed integer i and sufficiently large j. This talk is concerned with this question. In fact we study asymptotic behavior of $\operatorname{Ass}_R \operatorname{Ext}^i_R(R/\mathfrak{a}^j, M)$, $\operatorname{Supp}_R \operatorname{Ext}^i_R(R/\mathfrak{a}^j, M)$ or certain subsets of them.

A Characterization of Shellable And Sequentially Cohen-Macaulay Bipartite Graphs

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Let G be a simple undirected graph and Δ_G be a simplicial complex whose faces correspond to the independent sets of G. A graph G is called shellable if Δ_G is a shellable simplicial complex in the non-pure sense of Björner-Wachs. In this talk among the other results we give a characterization of shellable bipartite graphs.

On the Finiteness of Local Cohomology Modules

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Let R be a commutative Noetherian local ring of dimension d, $\mathfrak{a}$ an ideal of R, and M, N two finitely generated R-modules. We prove that if $d \leq 2$, then $\operatorname{Ext}^p_R(M, H^q_{\mathfrak{a}}(N))$ is $\mathfrak{a}$-cofinite for all $p, q \geq 0$. Also, if $d \leq 3$ then the set of associated primes of any quotient of

$$\operatorname{Ext}^p_R(R/\mathfrak{a}, H^q_{\mathfrak{a}}(M, N)) \quad \text{and} \quad \operatorname{Ext}^t_R(R/\mathfrak{a}, \operatorname{Ext}^p_R(M, H^q_{\mathfrak{a}}(N)))$$

are finite for all $p, q, t \geq 0$.

Projectivity of S-Posets

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In this talk, taking S to be a partially ordered monoid, we consider S-posets; that is, posets A on which there is an action from S which is compatible with the order of A and S. Then we characterize several kinds of epimorphisms in the category of S-posets. Finally, we discuss projectivity of S-posets with respect to that epimorphisms.

Some Results on Prime and Semiprime Submodules of Multiplication Modules

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Let R be a commutative ring. An R-module M is called a multiplication module if for each submodule N of M, $N = IM$, for some ideal I of R. In this paper, first we define the notion of residual of a submodule in M and then we obtain some related results. In particular, we state and prove some equivalent conditions for prime and semiprime submodules of a multiplication module.
Mathematics Subject Classification: 13C05, 13C13, 13A15
Key Words: Multiplication module; Prime submodule; Semiprime submodule.

A Generalized Principal Ideal Theorem for Modules

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The subject of this talk is an extended version of the Generalized Principal Ideal Theorem for modules.

Cominimaxness and Local Cohomology Modules

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An R-module M over a commutative ring R, is said to have *finite Goldie dimension*, if M does not contain an infinite direct sum of non-zero submodules, and M is said to have *finite* $\mathfrak{a}$*-relative Goldie dimension* if the Goldie dimension of the $\mathfrak{a}$-torsion submodule $\Gamma_{\mathfrak{a}}(M) :=$

$\bigcup_{n\geq 1}(0 :_M \mathfrak{a}^n)$ of M is finite. Also, we say that an R-module M is $\mathfrak{a}$-*cominimax* if the support of M is contained in $V(\mathfrak{a})$ and $\mathrm{Ext}^i_R(R/\mathfrak{a}, M)$ is $\mathfrak{a}$-minimax for all $i \geq 0$. In this talk, among other things, we show that:

Theorem. *Let $\mathfrak{a}$ be an ideal of R and let M be an R-module. Let t be a non-negative integer such that $\mathrm{H}^i_{\mathfrak{a}}(M)$ is $\mathfrak{a}$-cominimax for all $i < t$, and $\mathrm{Ext}^t_R(R/\mathfrak{a}, M)$ is $\mathfrak{a}$-minimax. Then for any $\mathfrak{a}$-minimax submodule N of $\mathrm{H}^t_{\mathfrak{a}}(M)$ and for any finitely generated R-module L with $\mathrm{Supp}\, L \subseteq V(\mathfrak{a})$, the R-module $\mathrm{Hom}_R(L, \mathrm{H}^t_{\mathfrak{a}}(M)/N)$ is $\mathfrak{a}$-minimax.*

Some Conjectures on Ext-indices

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In this talk, we are mainly concerned with the generalization of a question of Huneke and Jorgensen about localizations of AB rings. In this direction, we investigate the finiteness of Ext-indices for certain ring extensions. So we introduce some conjectures and discuss the relationships among them. We are also able to prove these conjectures in some special cases.

Tensor Product of Complexes

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Let R be a commutative Noetherian ring and let $\{a_1, \cdots, a_n\}$ be a set of $n \geq 1$ elements of R. Let $\mathfrak{a} = (a_1, \cdots, a_n)$ be the ideal of R that is generated by $\{a_1, \cdots, a_n\}$. Let M be an R-module and x a nonzero divisor of M. It is shown that $H^i(C^\cdot(a_1, \cdots, a_n, M) \otimes K^\cdot(x)) \cong H^{i-1}_{\mathfrak{a}}(M/xM)$, for all $i \geq 1$. In addition, it is shown that $H^i_{Rx+\mathfrak{a}}(M)$ is $Rx + \mathfrak{a}$-cofinite, if $H^i_{Rx+\mathfrak{a}}(M)$ is $\mathfrak{a}$-cofinite.

On the Regularity of Local Cohomology of Bigraded Algebras

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The Hilbert functions and the regularity of the graded components of local cohomology of a bigraded algebra are considered. Explicit bounds for these invariants are obtained for bigraded hypersurface rings.

Minima Prime Ideals and Semistar Operations

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Let R be a commutative integral domain and let $\star$ be a semistar operation of finite type on R, and I be a quasi-$\star$-ideal of R. We show that, if every minimal prime ideal of I is the radical of a $\star$-finite ideal, then the set $\mathrm{Min}(I)$ of minimal prime ideals over I is finite.

On Flat C(X)-Modules

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It is well known that a module over a principal ideal domain is flat if and only if it is torsion-free. Rings of continuous functions are not domains, so that to obtain similar flatness criteria for these rings, it is necessary to redefine the concept of torsion-free module. This article is devoted to the study of its property of rings of continues function.

Gorenstein Divisors

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Let X be a noetherian scheme which is Gorenstein in codimension one and satisfies the condition S_2 of Serre. In this talk, we introduce and study a new class of divisors, called Gorenstein divisors, both over schemes and also in affine case, over commutative noetherian rings. They are nondegenerate fractional ideals which are totally reflexive, in the sense of Auslander. This class of divisors fits well between the class of Cartier divisors and the class of generalized divisors, introduced and studied by Hartshorne. We show that over Gorenstein schemes effective Gorenstein divisors are in one-to-one correspondence with the Cohen-Macaulay closed subschemes of pure codimension one with no embedded points. Moreover, if scheme is Gorenstein of dimension less than or equal to two, then any generalized divisor is Gorenstein. We study the behavior of this class of divisors in certain cases. The talk is based on a joint work with J. Asadollahi and F. Jahanshahi.

On the Local Cohomology and Support for Triangulated Categories

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Recently a notion of support and a construction of local cohomology functors for [TR5] compactly generated triangulated categories is introduced and studied by Benson, Iyengar and Krause. Following their idea, we assign to any object of the category a new subset of Spec(R), again called the (big) support. We study this support and show that it satisfies axioms like exactness, orthogonality and separation. Using this support, we study the behavior of the local cohomology functors and show that these triangulated functors respect boundedness. Then, we restrict our study to the categories generated by only one compact object. This condition will able us to get some nice results. Our results show that one can get a satisfactory version of the local cohomology theory in the setting of triangulated categories, compatible with the known results for the local cohomology for complexes of modules. The talk is based on a joint work with J. Asadollahi and Sh. Salarian.

The Absolute Galois Group and Graphs on Surfaces

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In his Dessin d'Enfants project, Grothendieck suggested using graphs on surfaces as a geometric object on which the absolute Galois group G acts. The idea is that the combination of this action and the natural action of G on number fields should lead to new insights into the absolute Galois group and number theory. Belyi's theorem relating the arithmetic property of an algebraic curve being defined over a number field and the existence of a meromorphic function with only three critical values is a cornerstone of Grothendieck's program. I this lecture some features of this program and a class of graphs and certain related finite groups will be discussed.

Extremal Betti Numbers of Filtered Modules over a Local Regular Ring

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Let $(R, \mathfrak{n})$ be a regular local ring and let M be a finitely generated R-module. Given an $\mathfrak{n}$-stable filtration $\mathbb{M}$ of M, the Betti numbers of M are compared with those of the associated graded module $gr_{\mathbb{M}}(M)$.

The central result of the present talk says that if the minimal number of generators of M (as R-module) and the minimal number of generators of $gr_{\mathbb{M}}(M)$ (as $P = gr_{\mathfrak{n}}(R)$-module) coincide, then

$\beta_i(M) = \beta_i(gr_{\mathbb{M}}(M))$ for every $i \geq 0$ provided $gr_{\mathbb{M}}(M)$ is a componentwise linear module.

We apply the main result to the classical case of the local ring. Let I be an ideal of a local regular ring $(R, \mathfrak{n})$ and let $A = R/I$ with maximal ideal $\mathfrak{m}$. One has $gr_{\mathfrak{m}}(A) \simeq P/I^*$, where I^* is the homogeneous ideal of P generated by the initial forms of the elements of I. If we consider the ideal I equipped with the $\mathfrak{n}$-filtration $\mathbb{M} = \{I \cap \mathfrak{n}^i\}$ we have $gr_{\mathbb{M}}(I) = I^*$. So we can compare the resolution of I^* as P-module and the resolution of I as R-module.

As a consequence we show that if $\mu(I) = \mu(I^*)$ and I^* is componentwise linear, then the numerical invariants of a minimal free resolution of A and those of $gr_{\mathfrak{m}}(A)$ coincide. In particular, under these assumptions, depth $A =$ depth $gr_{\mathfrak{m}}(A)$.

The following corollaries are some applications of the above result:

1) Let $\mathrm{Gin}(I) := \mathrm{Gin}(I^*)$. For every $i \geq 0$ we have $\beta_i(I) = \beta_i(\mathrm{Gin}(I))$ if and only if $\beta_0(I) = \beta_0(\mathrm{Gin}(I))$.

2) Conca, Herzog and Hibi proved an upper bound for the Betti numbers of the local ring A in terms of the so-called generic annihilators of A. Under the assumption of the main result, the extremal Betti numbers are achieved.

3) Let $I \subseteq \mathfrak{n}^2$ be a non zero ideal of R. The Symmetric algebra of the maximal ideal $\mathfrak{m}$ of $A = R/I$ is $S_A(\mathfrak{m}) \simeq \oplus_{i \geq 0} \mathfrak{n}^i / I\mathfrak{n}^{i-1}$. Assume that $\mu(I) = \mu(Gin(I))$. If depth $A > 0$, then

$$\text{depth } S_A(\mathfrak{m}) \geq \text{depth } A + 1.$$

If A is Cohen-Macaulay, then depth $S_A(\mathfrak{m}) = \dim\ A + 1$. The talk is based on a joint work with M. E. Rossi.

A Note on Quasi-Gorenstein Rings

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In this paper, after giving a criterion for a Noetherian local ring to be quasi-Gorenstein, we obtain some sufficient conditions for a quasi-Gorenstein ring to be Gorenstein. In the course, we provide a slight generalization of a theorem of Evans and Griffith.

On the Depth of Modules of the form J/I

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In this talk we generalized the results in [4] from S/I to J/I, here $I \subset J$ are monomial ideals in $S = K[x_1, \dots, x_n]$. We give a combinatorial description of depth of a module of the form J/I and like in [4] we apply these techniques to show that Stanley's conjecture on Stanley decompositions holds for any module of the from J/I provided it holds for all Cohen–Macaulay module of the form J/I.

Let Δ be a simplicial complex of dimension $d-1$ on the vertex set $[n] = \{1, \dots, n\}$, K a field and $K[\Delta]$ the Stanley–Reisner ring of Δ. The depth of $K[\Delta]$ can be expressed in terms of the skeletons of Δ, as has been shown by D. Smith [7, Theorem 3.7] for pure simplicial complexes, and by Hibi [6, Corollary 2.6] in general. The jth skeleton of Δ is the simplicial subcomplex $\Delta^{(j)} = \{F \in \Delta \ |F| \leq j\}$ of Δ. The result is that $\operatorname{depth} K[\Delta] = \max\{j \ \Delta^{(j)}$ is Cohen–Macaulay$\}$.

The purpose of this paper is to generalize this result as follows: first note that we have the following chain of Stanley–Reisner ideals $I_\Delta = I_{\Delta^d} \subset I_{\Delta^{d-1}} \subset \cdots \subset I_{\Delta^0} \subset S$ with $\dim S/I_{\Delta^{(j)}} = j$ for all j. Now for $I \subset J$ we want to define in a natural way a similar chain of monomial ideals $I = I^{(d)} \subset I^{(d-1)} \subset \cdots \subset I^{(0)} \subset J$ with $\dim J/I^{(j)} = j$ for all j, and of course this chain should satisfy the condition that $\operatorname{depth} J/I = \max\{j \ \ J/I^{(j)}$ is Cohen–Macaulay$\}$. We show in the first part of talk that such a natural chain of monomial ideals with these properties indeed exists. In the case of S/I, the ideal $I^{(j)}$ was called in [4] the jth *skeleton ideal* of I.

For the construction of the ideals $I^{(j)}$ we consider the so-called *characteristic poset* $P^g_{J/I}$ introduced in [5]. Here $g \in \mathbb{N}^n$ is an integer vector such that $g \geq a$ for all a for which x^a belongs to the minimal set of monomial generators of I, and $P^g_{J/I}$ is the (finite) poset of all $b \in \mathbb{N}^n$ such that $b \leq g$ and $x^b \in J \setminus I$. All inequalities are understood to be componentwise. In case of a Stanley–Reisner ideal I_Δ and $g = (1, 1, \ldots, 1)$ this poset is just the face poset of Δ. For each $b \in \mathbb{N}^n$, let $\rho(b) = |\{j \; b(j) = g(j)\}|$. It has been shown in [5, Corollary 2.6] that $\dim J/I = \max\{\rho(b) \; b \in P^g_{J/I}\}$. We use this integer function ρ to define the ideals $I^{(j)}$, and let $I^{(j)}$ be the monomial ideal generated by I and all x^b $(b \in P^g_{J/I})$ with $\rho(b) > j$. It is the easy to see that $\dim J/I^{(j)} = j$ for all j. The crucial result however is that for all j, $I^{(j-1)}/I^{(j)}$ is Cohen-Macaulay module of dimension j. From this result we easily deduce a generalization of the result of Hibi, namely that $\operatorname{depth} J/I = \max\{j \; J/I^{(j)}$ is Cohen–Macaulay$\}$.

Then we apply the results and techniques introduced in first part to deduce some results on Stanley decomposition. Let M be a finitely generated $\mathbb{Z}^n$-graded S-module, $m \in M$ be a homogeneous element and $Z \subset X = \{x_1, \ldots, x_n\}$. We denote by $mK[Z]$ the K-subspace of M generated by all homogeneous elements of the form mu, where u is a monomial in $K[Z]$. The K-subspace $mK[Z]$ is called a *Stanley space of dimension* $|Z|$ if $mK[Z]$ is a free $K[Z]$-module.

A decomposition $\mathcal{D}$ of M as a finite direct sum of Stanley spaces is called a *Stanley decomposition* of M. The minimal dimension of a Stanley space in the decomposition $\mathcal{D}$ is called the *Stanley depth* of $\mathcal{D}$, denoted sdepth$\mathcal{D}$. We set

$$\operatorname{sdepth} M = \max\{\operatorname{sdepth}\mathcal{D} \; \mathcal{D} \text{ is a Stanley decomposition of } M\},$$

and call this number the *Stanley depth* of M. A famous conjecture of Stanley asserts that $\operatorname{sdepth} M \geq \operatorname{depth} M$.

We show that if $I \subset J$ are monomial ideals, then Stanley's conjecture on Stanley decompositions holds for J/I provided it holds whenever J/I is Cohen–Macaulay.

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After giving a brief history of research in commutative algebra in Iran, we give some information on the research interests in the work-group "commutative algebra" at the school of Mathematics at IPM.

Fixed Point Subalgebras of $(R, \Lambda)-$Graded Lie Algebras

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We study the subalgebra of fixed points of a root graded Lie algebra under a certain class of finite order automorphisms. As the centerless

core of extended affine Lie algebras or equivalently irreducible centerless Lie tori are examples of root graded Lie algebras, our work is an extension of some recent result about the subalgebra of fixed points of a Lie torus under a certain finite order automorphism.

Projective Modules and Projection Matrices

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Let R be a commutative ring with unity. For a finitely generated projective R-module M, it corresponds a projection (i.e., idempotent) matrix P. Conversely, the image of any projection matrix is a finitely generated projective module. The correspondence extends to the homomorphisms of finitely generated projective R-modules. This fact is recalled by H. Lombardi and C. Quitte in their recent paper [Seminormal rings (following Thierry Coquand), *Theoretical Computer Science* **392** (2008) 113-127], and is referred as an equivalence between the category of finitely generated projective R-modules and the category of projection matrices. However, to establish the equivalence of categories, one needs to enlarge the first category to the category of triples (M, R^m, f) where M is a direct summand of R^m and $f : R^m \longrightarrow M$ is the canonical projection. The equivalence of these categories provides a dictionary to explain certain properties of finitely generated projective modules in the language of projection matrices. For example, the tensor product of two finitely generated projective R-modules corresponds to the Kronecker product of their projection matrices and the wedge products of these R-modules can be explained by the ideals of minors of the corresponding projection matrices. More significantly, the corresponding statement for isomorphisms of such R-modules can nicely be explained in light of the category of triples. By the celebrated Serre conjecture proved by Quillen and Suslin, for $R = k[x_1, \cdots, x_n]$, the ring of polynomials over a field k, any projective R-module is free. Thus, it is important to know when a finitely generated projective R-module is free. Accordingly, it is challenging to use the dictionary for the equivalent property in terms of matrices.

Homomorphisms of Simplicial Complexes

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In this talk a definition for a homomorphism between two simplicial complexes will be presented. Then, adaptability of this homomorphism with algebraic homomorphisms of Stanley-Reisner rings associated to the complexes will be checked.

Local Cohomology Modules with respect to an Ideal Containing the Irrelevant Ideal

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Let $R = \bigoplus_{n\geq 0} R_n$ be a homogeneous Noetherian ring, let M be a finitely generated graded R-module and let $R_+ = \bigoplus_{n>0} R_n$. Let $\mathfrak{b} := \mathfrak{b}_0 + R_+$, where $\mathfrak{b}_0$ is an ideal of R_0. In this talk, we first study the finiteness and vanishing of the n-th graded component $\mathrm{H}^i_{\mathfrak{b}}(M)_n$ of the i-th local cohomology module of M with respect to $\mathfrak{b}$. Then, among other things, we show that the set $\mathrm{Ass}_{R_0}(\mathrm{H}^i_{\mathfrak{b}}(M)_n)$ becomes ultimately constant, as $n \to -\infty$, in the following cases:

(i) $\dim(\frac{R_0}{b_0}) \leq 1$ and $(R_0, \mathfrak{m}_0)$ is a local ring;
(ii) $\dim(R_0) \leq 1$ and R_0 is either a finite integral extension of a domain or essentially of finite type over a field;
(iii) $i \leq g_{\mathfrak{b}}(M)$, where $g_{\mathfrak{b}}(M)$ denotes the cohomological finite length dimension of M with respect to $\mathfrak{b}$.

Also, we establish some results about the Artinian property of certain submodules and quotient modules of $\mathrm{H}^i_{\mathfrak{b}}(M)$.