Incidence Energy

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Energy

- (*Gutman, 1978*) The energy of a graph *G* is defined as

$$E(G) = \sum_{i=1}^{n} |\lambda_i(G)|$$

- The singular values of a matrix *A* are the square roots of the eigenvalues of *AA** and is denoted by $\sigma_1(A) \geq \cdots \geq \sigma_n(A)$. If *A* is a hermitian matrix, then the singular values of *A* are the absolute values of its eigenvalues.

Incidence Energy

- (*Nikiforov, 2007*) For any matrix A call the value $E(A)=\sum_i \sigma_i(A)$ the energy of A.
- Let $\sigma_1(G),\cdots,\sigma_n(G)$ be the singular values of the incidence matrix of a graph G, the quantity $IE(G)=\sum_i \sigma_i(G)$ is called the ***incidence energy*** of the graph G.

Example

- $\mathbb{E}(S_n) = n - 2 + \sqrt{n}.$

Energy & Incidence Energy

- $E(G) \geq 0$, equality is attained iff G has no edges.
- $IE(G) \geq 0$, equality holds iff G has no edges.
- If the graph G consists of connected components $G_1, \ldots, G_c$, then

$$E(G) = \sum_i E(G_i), \quad IE(G) = \sum_i IE(G_i).$$

Energy & Incidence Energy

Theorem: Let G be a graph, then $\mathbb{E}(G)=\dfrac{E(\hat{G})}{2}$ *in which* $\hat{G}$ *is the bipartite graph with adjacency matrix*

$$\begin{pmatrix} 0 & I(G) \\ I(G)^t & 0 \end{pmatrix}.$$

The graph $\hat{G}$ is the bipartite graph which is obtained from G by adding a vertex on each edge of G.

Energy & Incidence Energy

- If the energy of a graph is rational, then it must be an even number.
- The incidence energy of a graph can not be an odd number.
- $E(A) \geq rank(A(G))$.
- $IE(A) \geq rank(I(G))$. Let G be any connected graph. If G is bipartite, $rank(I(G)) = n-1$ otherwise $rank(I(G)) = n$.

Energy & Incidence Energy

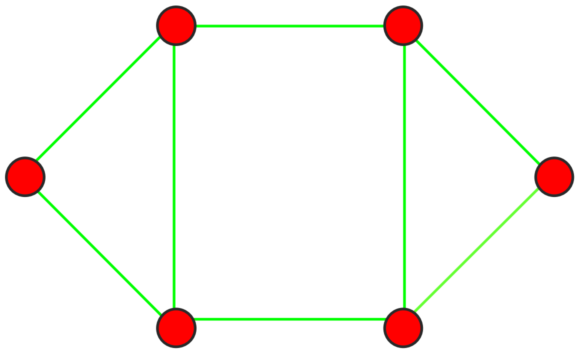
- Let G be a graph of order n with m edges. Then

$$\sqrt{2m} \leq E(A), IE(G) \leq \sqrt{2mn},$$

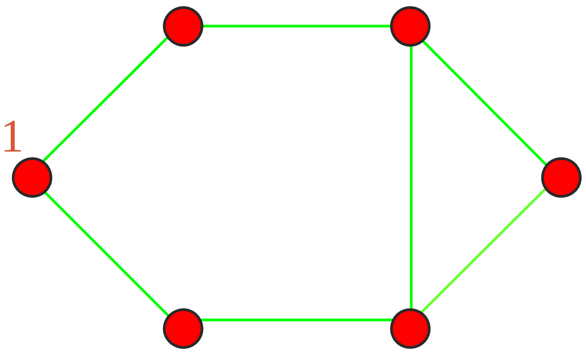
With left equality holding iff $m<2$, and right equality holding iff $m=0$.

Energy & Incidence Energy

H



H_1

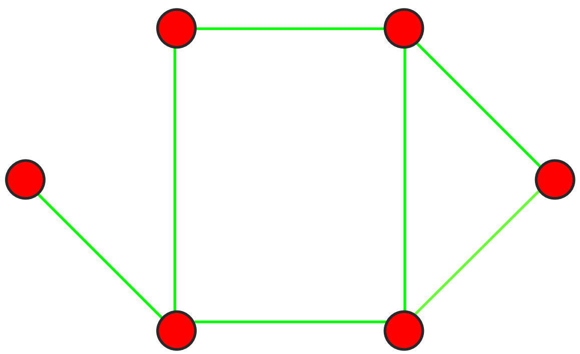


$$\{1+\sqrt{3}, \sqrt{2}, 1-\sqrt{3}, -\sqrt{2}, -2\}$$

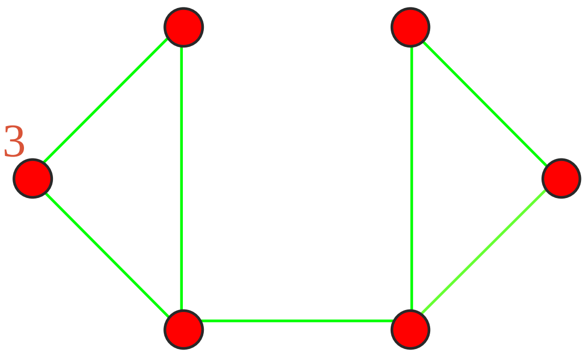
$$E \approx 8.2925$$

$$E(H_1) \approx 8.3898 > E(H)$$

H_2



H_3



$$E(H_2) \approx 7.7662 < E(H)$$

$$E(H_2) = E(H)$$

Energy & Incidence Energy

- Let H be an induced proper subgraph of a simple graph G. Then $E(G) \geq E(H)$.
- Let G be a graph and E be a non-empty subset of $E(G)$, then $IE(G) > IE(G \setminus E)$.
- Among all graphs with n vertices, the complete graph is the only graph with maximum incidence energy.

Energy & Incidence Energy

- Let e be any edge of G, then
  - If H is an induced subgraph of the graph G, then $E(G\backslash H)-E(H)\leq E(G)\leq E(G\backslash H)+E(H)$.
  - $\sqrt{IE(G\backslash\{e\})^2+2}\leq IE(G)\leq IE(G\backslash\{e\})+2.$

Energy & Incidence Energy

- Let T be a tree with n vertices, which is not path, then

$$E(T) \le E(P_n), \; IE(T) \le IE(P_n).$$

Any Question?