Instantons and Monopoles in Noncommutative Geometry

(3 Lectures)

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These lectures are devoted to recent work on the geometry of instantons and monopoles on noncommutative spaces, in particular spheres and projective spaces.

A first class of examples consists of noncommutative manifolds coming from deforming the action of a real tori. On a four-sphere of such a kind, there are noncommutative families (i.e. families parametrized by a noncommutative algebra) of instantons. Topological charges are computed by noncommutative index theorems.

A second class of examples comes from compact quantum groups. In particular, on the projective plane CPq2 there are both instanton bundles and monopole bundles. For these class of spaces, beside 'usual' indices of operators there are also so called 'quantum indices'. In contrast to the classical situation (and to the previous class of examples), by integrating powers of the curvatures one needs now 'twisted integrals' and the result is not an integer any longer but rather a so called q-integer which, in turn, can be obtained as a 'q-index' of a suitable Fredholm operator.