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The Structure of Perturbative Quantum Gauge Theories

(3 Lectures)

W. van Suijlekom
Radboud University Nijmegen
the Netherlands

In this series of 3 lectures we would like to give an overview of the physicists' approach to quantum gauge theories and present some of the mathematically rigorous results that can be obtained in this context.

Quantum gauge theories are the building blocks of the extremely well-tested Standard Model of high-energy physics. It describes all known elementary particles and their interactions. Mathematically, however, these theories are not so well understood, in sharp contrast to their classical counterparts. We try to unravel the structure of quantum gauge theories by taking a perturbative (i.e. formal power series) approach. In particular, we try to understand two aspects of quantum gauge theories: renormalization and quantum gauge (i.e. BRST) symmetries.

After an introduction to these two physical ideas, we describe them mathematically in terms of the Connes-Kreimer Hopf algebra and a Gerstenhaber algebra, respectively. Their connection is established by turning the latter into a comodule Gerstenhaber algebra over the former. This provides the right algebraic setup for a mathematical understanding of the compatibility between renormalization and quantum gauge symmetries. Finally, we hope to arrive at the connection between this algebraic setup and BRST-cohomology, which plays a central role in the usual approach to renormalization of quantum gauge theories.