k-Equal Subspace Arrangements Revisited

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The k-equal arrangement is the collection of subspaces in $\mathbb{R}_n$ given by equations of the form $x_{i_1} = x_{i_2} = \cdots = x_{i_k}$, over all indices $1 \leq i_1 < i_2 < \cdots < i_k \leq n$. In this talk, we describe the k-parabolic arrangement, a generalization of the k-equal arrangement for any finite real reflection group. When $k = 2$, these arrangements correspond to the well-studied Coxeter arrangements, including the Braid arrangement when W is of type A. In 1963, Fadell, Fox, and Neuwirth showed that the complement of the complex braid arrangement is a $K(\pi, 1)$ space, and that its fundamental group is isomorphic to the pure braid group. Brieskorn (1971) generalized the last result to complexified W Coxeter arrangements by showing that the fundamental group is isomorphic to the pure Artin group of type W. Khovanov (1996) gave a real counterpart to Fadell, Fox and Neuwirth's result when W is of type A (and B) by showing that the complement of the 3-equal arrangement (over $\mathbb{R}$) is a $K(\pi, 1)$ space, and by giving an algebraic description of its fundamental group. We generalize Khovanov's result and obtain an algebraic description of the fundamental group of the complement of the 3-parabolic arrangement for arbitrary finite reflection group. Our description is a real analogue of Brieskorn's one. We conjecture that for W of any type, the complement of the 3-parabolic arrangement is a $K(\pi, 1)$ space. This is joint work with Christopher Severs and Jacob White.