# Some generalizations of Cohen-Macaulay simplicial complexes

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**Abstract**

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Let $\Delta$ be a simplicial complex on a vertex set $V$.

a) Following Terai and Yanagawa, for an integer $r \geq 2$, $\Delta$ is said to be $S_r$ if $k[\Delta]$, the Stanley-Reisner ring of $\Delta$, satisfies the $S_r$ condition of Serre. We say that $\Delta$ is sequentially $S_r$ if $k[\Delta]$ is sequentially $S_r$ is the sense of Stanley. It appears that $\Delta$ is sequentially $S_r$ if and only if its pure skeletons are $S_r$. This generalizes a result of Duwal on sequentially Cohen-Macaulay complexes. On the other hand, $\Delta$ is sequentially $S_r$ if and only if the ideal of the Alexander dual of $\Delta$ is componentwise linear in the first $r$ steps. This generalizes a result of Terai and Yanagawa on $S_r$ complexes, and a result of Herzog and Hibi on sequentially Cohen-Macaulay complexes.

b) For a nonnegative integer $t$, we say that $\Delta$ is $\mathrm{CM}_t$ if for $\sigma \in \Delta$ with $\#\sigma \geq t$, $\mathrm{lk}_\Delta(\sigma)$ is Cohen-Macaulay. For positive integer $k$, $\Delta$ is called $k$-$\mathrm{CM}_t$ if for and $W \subset V$ with $\#W < k$, the complex $\Delta_{V \setminus W}$ is $\mathrm{CM}_t$. A $\mathrm{CM}_0$ complex is the same as a Cohen-Macaulay complex and a $\mathrm{CM}_1$ complex is the same as a Buchsbaum complex. We give some homological characterization for $\mathrm{CM}_t$ and $k$-$\mathrm{CM}_t$ complexes. We show that $\Delta$ is $k$-$\mathrm{CM}_t$ if $k$-$\mathrm{CM}_{t-1}$ for any nonempty $\sigma \in \Delta$. Furthermore, if $\Delta$ is $k$-$\mathrm{CM}_t$, then the submaximal skeleton of $\Delta$ is $(k+1)$-$\mathrm{CM}_t$. This generalizes a result of Hibi on Cohen-Macaulay complexes and a result of Miyazaki on Buchsbaum complexes.