The 8th Seminar on Commutative Algebra and Related Topics,
November 30 & December 1, 2011, IPM, Tehran

Derived Dimension of Artin Algebras

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Let $\mathcal{T}$ be a triangulated category. A notion of dimension for $\mathcal{T}$ has been introduced by Rouquier. Roughly speaking the dimension of $\mathcal{T}$, denoted $\dim \mathcal{T}$, measures the minimum steps that one need to construct $\mathcal{T}$ out of an object C. As the main application, Rouquier solved one of the old open problems related to the representation dimension of finite-dimensional artin algebras. In fact, he showed that the dimension of the triangulated category $\mathbf{D}^b(\text{mod-}\Lambda)$, bounded derived category of Λ, where Λ is a finite-dimensional artin algebra, is a lower bound for the representation dimension of Λ. Using this fact, he proved that any natural number, except 1, occurs as the representation dimension of some algebra and so there is no upper bound on the representation dimension of artin algebras. After this result, dimension of $\mathbf{D}^b(\text{mod-}\Lambda)$, also known as the derived dimension of Λ, has been studied by several mathematicians.

In this talk, which is based on a joint work with R. Hafezi, we will first recall the notions of the representation dimension of artin algebras and the Rouquier dimension of triangulated categories and then will focus on the derived dimension of the path algebra of a finite quiver and compare it to the derived dimension of the underlying algebra.

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On Gorenstein Homological Dimension of Groups

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The notion of Gorenstein flat dimension of modules over an associative ring has been defined by Enochs, Jenda and Torrecillas in to complete the analogy between classical homological dimension theory and Gorenstein homological dimension theory. Here, we focus on the Gorenstein flat dimension of modules over group rings. In particular, we consider the Gorenstein flat dimension of the trivial Γ-module $\mathbb{Z}$. This invariant, that will be called Gorenstein homological dimension of Γ, denoted $\operatorname{Ghd}\Gamma$, is finer than homological dimension of Γ and is tightly connected to the supremum of the flat lengths of injective Γ-modules, $\operatorname{sfli}\Gamma$. Namely, it is shown that $\operatorname{sfli}\Gamma$ is finite if and only if the Gorenstein flat dimension of any Γ-module is finite. Furthermore, it reflects some properties of the underlying group. For example, we show that a group Γ is finite if and only $\operatorname{Ghd}\Gamma = 0$. Also, it is proved an analogue of Serre's Theorem for this invariant,i.e. for a subgroup Γ' of Γ of finite index, $\operatorname{Ghd}\Gamma' = \operatorname{Ghd}\Gamma$, provided $\operatorname{silf}\Gamma$ is finite, whereas $\operatorname{silf}\Gamma$ is the supremum of the injective lengths of flat Γ-modules. We also examine the relation between the Gorenstein homological dimension and Ikenaga's generalized homological dimension of a group Γ, $\underline{\operatorname{hd}}\,\Gamma$. This invariant is defined as the supremum of those integers n, for which there exists a $\mathbb{Z}$-torsion-free Γ-module M and an injective Γ-module I, such that $\operatorname{Tor}_n^{\Gamma}(M, I) \neq 0$. With the aid of an example, we show that it may happen that both $\operatorname{Ghd}\Gamma$ and $\underline{\operatorname{hd}}\,\Gamma$ are finite, but they are not equal. These results based on a joint work with J. Asadollahi, A. Hajizamani and Sh. Salarian.

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Commutative Local Rings Whose Ideals are Direct Sums of Cyclic Modules

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A theorem from commutative algebra due to Köthe and Cohen-Kaplansky states that, "a commutative ring R has the property that every R-module is a direct sum of cyclic modules if and only if R is an Artinian principal ideal ring". Therefore, an interesting natural question of this sort is "*whether the same is true if one only assumes that every ideal is a direct sum of cyclic modules*?" More recently, this question was answered in [Journal of Algebra 345 (2011) 257-265] for the case R is a finite direct product of commutative Noetherian local rings. The goal of this paper is to answer this question in the case R is a finite direct product of commutative local rings (not necessarily Noetherian). The structure of such rings is completely described. In particular, it is shown that every ideal of a local ring $(R, \mathcal{M})$ is a direct sum of cyclic R-modules if and only if $\mathcal{M} = Rx \oplus Ry \oplus (\bigoplus_{\lambda\in\Lambda} Rw_\lambda)$ where Λ is an index set, Rw_λ's are simple R-modules and $R/\mathrm{Ann}(x)$, $R/\mathrm{Ann}(y)$ are principal ideal rings.

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What is Dawson's Chess?

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Recently E. Miller reposed an open question about Dawson's chess. This question is for almost 70 years known to be open. The aim of this talk is to introduce Dawson's chess in more details and provide sufficient arguments about its hardness. We introduce the notion of QUASI-DAWSON'S CHESS and prove that playing on a $3 \times d$ board, the first player is loser if and only if $d \mod 5 = 1$ or $d \mod 5 = 2$. We also design two algorithms that are responsible for storing the results of QUASI-DAWSON'S CHESS games having less than $d + 1$ files and finding the strategy that leads to win, if there is a possibility of winning. Moreover we show that the total complexity of our algorithms is $O(d^2)$. The implementations of our algorithm in `C++` admit the main results of the paper even for large values of d.

Linkage of Finite Gorenstein Dimension Modules

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The theory of linkage of algebraic varieties introduced by Peskine and Szpiro. Martsinkovsky and Strooker give its analogous definition for modules over non–commutative semiperfect Noetherian rings by using the composition of the two functors: transpose and syzygy. These functors and their compositions were studied by Auslander and Bridger. The Gorenstein (or G-) dimension was introduced by Auslander and studied by Auslander and Bridger. In this talk we study the theory of linkage for class of modules which have finite Gorenstein dimensions. In particular, for a horizontally linked module M of finite and positive G-dimension, we study the role of its reduced grade, r.grade$\,(M)$, on the depth of its linked module λM.

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On an Endomorphism Ring of Local Cohomology

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Let I be an ideal of a local ring $(R, \mathfrak{m})$ with $d = \dim R$. For the local cohomology module $H^i_I(R)$ it is a well-known fact that it vanishes for $i > d$ and is an Artinian R-module for $i = d$. In the case that the Hartshorne-Lichtenbaum Vanishing Theorem fails, that is $H^d_I(R) \neq 0$, we explore its fine structure. In particular, we investigate its endomorphism ring and related connectedness properties. In the case R is complete we prove - as a technical tool - that $H^d_I(R) \simeq H^d_{\mathfrak{m}}(R/J)$ for a certain ideal $J \subset R$. Thus, properties of $H^d_I(R)$ and its Matlis dual might be described in terms of the local cohomology supported in the maximal ideal. In recent research there is an interest in endomorphism rings of certain local cohomology modules $H^i_I(R)$. This was done in the case of $i = \dim R$ and $I = \mathfrak{m}$ by Hochster and Huneke and in the case of $i = \operatorname{height} I$ and R a Gorenstein ring. Here we continue with the case of $i = \dim R$ and an arbitrary ideal $I \subset R$. In particular we investigate the natural ring homomorphism

$$\hat{R} \to \operatorname{Hom}_{\hat{R}}(H^d_I(R), H^d_I(R)), \; d = \dim R.$$

We describe its kernel, characterize when it is an isomorphism, prove that the endomorphism ring $\operatorname{Hom}_{\hat{R}}(H^d_I(R), H^d_I(R))$ is commutative and decide when it is a local Noetherian ring. Note that in the case of $I = \mathfrak{m}$ we recover results shown by Hochster and Huneke.

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New Homological Invariants for Modules over Group Rings

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Recently the notions of sfliΓ, the supremum of the at lengths of injective Γ-modules, and silfΓ, the supremum of the injective lengths of at Γ-modules has been studied by some authors. These homological invariants are based on spli and silp invariants of Gedrich and Gruenberg and it is shown that they have enough potential to play an important role in studying homological conjectures in cohomology of groups. In this paper we will study these invariants. It turns out that, for any group $\Gamma < \infty$, the finiteness of silfΓ implies the finiteness of sfliΓ, but the converse is not known. We investigate the situation in which sfli$\Gamma < \infty$ implies silf$\Gamma < \infty$. The statement holds for example, for groups Γ with the property that at Γ-modules have finite projective dimension.

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Prime Submodules and Spectral Spaces

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We establish conditions for Spec(M) to be Noetherian and spec-tral space, with respect to the different topologies. We used rings with Noetherian spectrum to produce plentiful examples of modules with Noetherian spectrum that have not appeared in the literature previously. In particular, we show that every Z-module has Noetherian spectrum. Another main subject of this paper is presentation of conditions under which a module is top. In particular, we show that every distributive module is top, every content weak multiplication R-module M is also top, and moreover, if R has Noetherian spectrum, then Spec(M) is a spectral space.

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Tame Loci of Some Graded Modules

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Let M be a finitely generated graded module over a Noetherian homogeneous ring $R = \bigoplus_{n \in \mathbb{N}_0} R_n$. For each $i \in \mathbb{N}_0$ let $H^i_{R_+}(M)$ denote the i-th local cohomology module of M with respect to the irrelevant ideal $R_+ = \bigoplus_{n>0} R_n$ of R, furnished with its natural grading. We study the tame loci $\mathfrak{T}^i(M)^{\leq 3}$ at level $i \in \mathbb{N}_0$ in codimension ≤ 3 of M, that is the sets of all primes $\mathfrak{p}_0 \subset R_0$ of height ≤ 3 such that the graded $R_{\mathfrak{p}_0}$-modules $H^i_{R_+}(M)_{\mathfrak{p}_0}$ are tame.

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Local Cohomology Modules and Derived Functors

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Some part of this talk is a joint work with Dr. F. Khosh-Ahang.

In this talk we provide some isomorphisms of derived functors and local cohomology modules. Then we give some applications of these isomorphisms in endomorphisms of local cohomology modules.

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The Role of the Syzygies of Local Cohomology Modules

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Suppose that R is a commutative Noetherian ring with non-zero identity. The main goal of this note is characterizing left and right derived functors of special local cohomology modules (or their Matlis duals) by means of left and right derived functors (or Matlis duals) of $\frac{R}{(\underline{x})}$, where $\underline{x} := x_1, \ldots, x_n$ is a standard system of parameters for R. In this way, we exploit of the relations between their syzygies.

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Local Homology and Gorenstein Flat Modules

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Let R be a commutative Noetherian ring, $\mathfrak{a}$ an ideal of R and $\mathcal{D}(R)$ denote the derived category of R-modules. We investigate the theory of local homology in conjunction with Gorenstein flat modules. Let X be a homologically bounded to the right complex and Q a bounded to the right complex of Gorenstein flat R-modules such that Q and X are isomorphic in $\mathcal{D}(R)$. We establish a natural isomorphism $\mathbf{L}\Lambda^{\mathfrak{a}}(X) \simeq \Lambda^{\mathfrak{a}}(Q)$ in $\mathcal{D}(R)$ which immediately asserts that $\sup \mathbf{L}\Lambda^{\mathfrak{a}}(X) \leq \operatorname{Gfd}_R X$. This isomorphism yields several consequences. For instance, in the case R possesses a dualizing complex, we show that $\operatorname{Gfd}_R \mathbf{L}\Lambda^{\mathfrak{a}}(X) \leq \operatorname{Gfd}_R X$. Also, we establish a criterion for regularity of Gorenstein local rings.

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Aluffi Torsion-free Ideals

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A special class of algebras which are intermediate between the symmetric and the Rees algebras of an ideal was introduced by P. Aluffi. His purpose was to describe the characteristic cycle of a hypersurface, parallel to well known conormal cycle in intersection theory. A. Nasrollah Nejad and A. Simis called such an algebra the *Aluffi algebra*. Given a commutative ring R and ideals $J \subset I \subset R$, the Aluffi algebra of I/J is defined by $\mathcal{A}_{R/J}(I/J) := \mathcal{S}_{R/J}(I/J) \otimes_{\mathcal{S}_R(I)} \mathcal{R}_R(I)$. The Aluffi algebra is squeezed as $\mathcal{S}_{R/J}(I/J) \twoheadrightarrow \mathcal{A}_{R/J}(I/J) \twoheadrightarrow \mathcal{R}_{R/J}(I/J)$ and is moreover a residue ring of the ambient Rees algebra $\mathcal{R}_R(I)$. The kernel of the right hand surjection is called the module of Valabrega-Valla which is the torsion of the Aluffi algebra. We call a pair of ideals $J \subset I$, *Aluffi torsion-free* if the surjection $\mathcal{A}_{R/J}(I/J) \twoheadrightarrow \mathcal{R}_{R/J}(I/J)$ is injective. The goal of this talk is to find some examples of Aluffi torsion-free pairs which are in the main streams of researches in Commutative Algebra and Algebraic Geometry. We classify completely the ideals generated by 2-minors of a $2 \times n$ matrix of linear forms and edge ideals of graphs in terms of the Aluffi torsion-free property. The talk is based on joint work with Rashid Zaare-Nahandi.

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Sequentially Cohen–Macaulay Modules with Respect to an Irrelevant Ideal

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In this talk, for a finitely generated bigraded S-module M where S is a standard bigraded polynomial ring we define the relative Cohen–Macaulay filtration $\mathcal{F}$ of M with respect to the bigraded irrelevant ideal Q. We call M to be sequentially Cohen–Macaulay with respect Q if M admits a relative Cohen–Macaulay filtration with respect to Q. We investigate the algebraic properties of these modules and compute the length of a relative Cohen–Macaulay filtration with respect to Q explicitly. All hypersurface rings that are sequentially Cohen–Macaulay with respect to Q are classified.

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Path Ideals of Graphs

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Let $G = (V, E)$ be a finite simple graph with vertex set $V = \{x_1, \ldots, x_n\}$ and edge set E. Associated to the graph G is a monomial ideal

$$I(G) = (x_i x_j : \{x_i, x_j\} \in E),$$

in the polynomial ring $R = k[x_1, \ldots, x_n]$ over a field k, called the **edge ideal** of G.
The path ideal of a graph was first introduced by Conca and De Negri. Fix an integer $2 \leq t \leq n$ and let G be a directed graph. A sequence $x_{i_1}, \ldots, x_{i_t}$ of distinct vertices, is called a **path** of length t if there are $t-1$ distinct directed edges $e_1, \ldots, e_{t-1}$ where e_j is a directed edge from x_{i_j} to $x_{i_{j+1}}$. Then the **path ideal** of G of length t is the monomial ideal

$$I_t(G) = (x_{i_1} \cdots x_{i_t} : x_{i_1}, \ldots, x_{i_t} \text{ is a path of length t in G}).$$

We have $I_2(G) = I(G)$, thus the path ideal is also called the **generalized edge ideal** of G.
In this talk, we investigate some algebraic properties of the path ideals of some classes of graphs, which have been studied in the case of edge ideals. Actually, we characterize all unmixed and Cohen-Macaulay path ideals of these graphs. Moreover, we determine when such these ideals satisfy the Serre's condition S_r.
This talk is based on a joint work with D. Kiani.

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What Pullback Construction Can Do for You

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In this talk I want to review pullback construction and show via some examples that, why pullback constructions are important in multiplicative ideal theory.

Binomial Edge Ideals of Some Classes of Graphs

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Let G be a simple graph on the vertex set $[n]$ and $R = K[x_1, \ldots, x_n, y_1, \ldots, y_n]$ be the polynomial ring over the field K. The *binomial edge ideal* of G is the ideal

$$J_G = (f_{ij} : \ \{i,j\} \in E(G) \text{ and } i < j) \subset R,$$

where $f_{ij} = x_i y_j - x_j y_i$. The goal of this talk is to compute the numerical invariants of the binomial edge ideals of graphs like depth and Hilbert function.

This talk is based on a joint work with F. Mohammadi

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Bounds for Dimensions of Derived and Singularity Categories

Ryo Takahashi
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The notion of the dimension of a triangulated category has been introduced by Bondal, Rouquier and Van den Bergh. In this talk, for a commutative Noetherian ring R we study upper/lower bounds for the dimensions of the bounded derived category of finitely generated R-modules and the singularity category of R.

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A New Method to Check Cohen-Macaulayness of Bipartite Graphs

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In this note, we give some combinatorial conditions for a bipartite graph which are equal to Cohen-Macaulayness of the graph. These conditions are not depending to any ordering on vertices of the graph and we may derive a fast algorithm to check Cohen- Macaulayness of a given bipartite graph. Also several other properties of such graphs are presented as corollaries of the main theorem.