Tame loci of some graded modules

Maryam Jahangiri (Joint work with M. Brodmann)

Tarbiat Moallem University

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Preliminaries

Definition

Let $R = \bigoplus_{n \geq 0} R_n$ be a standard graded ring, i.e. *$R = R_0[l_1, \cdots, l_t]$ where $l_1, \cdots, l_t \in R_1$, and $X = \bigoplus_{n \in \mathbb{Z}} X_n$ be a graded R-module. Then, X is said to be Tame (or asymptotically gap free), if*

$$\text{either } X_n = 0 \text{ for all } n \ll 0 \text{ or else } X_n \neq 0 \text{ for all } n \ll 0.$$

Example

Any Notherian or Artinian graded R-module is Tame.

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Let $R = \bigoplus_{n\geq 0} R_n$ be a standard graded ring, $R_+ = \bigoplus_{n>0} R_n$ be the irrelevant ideal of R and let $M = \bigoplus_{n\in\mathbb{Z}} M_n$ be a non-zero finitely generated graded R-module.

Remark

It is wellknown that for each $i \in \mathbb{N}_0$, $H^i_{R_+}(M)$, the ith local cohomology modules of M with respect to R_+, has a natural grading and that $H^i_{R_+}(M)_n = 0$ for sufficiently large values of $n \in \mathbb{Z}$.
But we know not much about the graded components $H^i_{R_+}(M)_n$ for sufficiently small values of $n \in \mathbb{Z}$ and the asymptotic behavior of $H^i_{R_+}(M)_n$ when $n \to -\infty$ attracts lots of interest.

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Outline

Question

(M. Brodmann)

Is the graded R-module $H^i_{R_+}(M)$ Tame?

Example

(Chardin, Cutkosky, Herzog, Srinivasan: 2008)
There is a Rees algebra R of dimension 4 over a 3 dimensional local domain R_0 such that the graded R - module $H^2_{R_+}(R)$ is not tame!

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History

$H^i_{R_+}(M)$ is Tame in the following cases:

$dim(R_0)$	condition on R_0	condition on M
0	-	-
1	semilocal	-
1	finite integral extension of a domain	-
1	e. f.t.f.	-
1	-	Cohen-Macaulay
2	semilocal	-
2	domain and e. f. t. f.	torsion free over R_0
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Problem

Is the set

$$\mathfrak{T}^i_M := \{\mathfrak{p}_0 \in Spec(R_0) | H^i_{R_+}(M)_{\mathfrak{p}_0} \text{ is Tame}\}$$

(the ith cohomological Tame loci of M)
open in $Spec(R_0)$ with Zariski topology?

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Results

Proposition

If the set $minAss_{R_0}(H^i_{R_+}(M)_n)$ *is asymptotically stable for* $n \ll 0$ *(i.e.* $\exists X \subseteq Spec(R_0)$ *such that* $minAss_{R_0}(H^i_{R_+}(M)_n) = X$ *for all* $n \ll 0$*), then* $\mathfrak{T}^i_M = Spec(R_0)$*.*

Corrolary

$\mathfrak{T}^j_M = Spec(R_0)$ *in the following cases:*
j is the least integer for which $H^j_{R_+}(M)$ *is not finitely generated;*
j is the last non-vanishing level of $H^j_{R_+}(M)$*.*

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If the set $minAss_{R_0}(H^i_{R_+}(M)_n)$ *is asymptotically stable for* $n \ll 0$ *(i.e.* $\exists X \subseteq Spec(R_0)$ *such that* $minAss_{R_0}(H^i_{R_+}(M)_n) = X$ *for all* $n \ll 0$*), then* $\mathfrak{T}^i_M = Spec(R_0)$.

Corrolary

$\mathfrak{T}^j_M = Spec(R_0)$ *in the following cases:*
j is the least integer for which $H^j_{R_+}(M)$ *is not finitely generated;*
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Results

For $X \subseteq Spec(R_0)$ and $t \in \mathbb{N}$ set $X^{\leq t} := \{\mathfrak{p}_0 \in X \mid ht(\mathfrak{p}_0) \leq t\}$.

Lemma

Let R_0 be essentially of finite type over a field. Then $Ass_{R_0}(H^i_{R_+}(M)_n)^{\leq 2}$ is asymptotically stable for $n \ll 0$.

Corrolary

Let R_0 be essentially of finite type over a field. Then for all $i \in \mathbb{N}$, $(\mathfrak{T}^i_M)^{\leq 2} = Spec(R_0)^{\leq 2}$.

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Theorem

Let R_0 be a domain which is essentially of finite type over a field and let M be torsion free over R_0. Then For all $i \in \mathbb{N}$, $(\mathfrak{T}^j_M)^{\leq 3}$ is open and dense in $Spec(R_0)^{\leq 3}$.

Theorem

Let $dim(H^{i-1}_{R_+}(M)) \leq 1$ and $dim(H^{i-2}_{R_+}(M)) \leq 2$. Then $(\mathfrak{T}^i_M)^{\leq 3} = Spec(R_0)^{\leq 3}$.

Example

There exists a finitely generated graded module M over a standard graded ring with a 4 dimensional domain base ring, for which $(\mathfrak{T}^j_M)^{\leq 4}$ is not open in $Spec(R_0)^{\leq 4}$.

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