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Tame hereditary algebras

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Abstract

Let k be an algebraically closed field and let Q be a finite quiver without oriented cycles. By $H = kQ$ we denote the path algebra of Q, which is automatically *hereditary*, that is, has global dimension at most one. Such an algebra H, as any finite dimensional algebra, is either *representation-finite*, that is, admits only finitely many (isomorphism classes of) indecomposable finite dimensional modules, or it it is *tame*, that is, the finite dimensional indecomposable modules can be arranged in an explicit, infinite 'list', or else it is *wild* meaning that it is hopeless to expect a complete classification of all finite dimensional indecomposable modules. By a theorem of Gabriel and subsequent work of Dlab and Ringel, it is well known that H is representation-finite if and only if the underlying graph of the quiver Q is a *Dynkin diagram*; further H is tame if and only if Q has *extended Dynkin type*, and H is wild otherwise where Q is neither Dynkin nor extended Dynkin.

The tame hereditary algebras form the entry door to the study of algebras of infinite representation type in general. In my series of lectures, I will discuss the classification of indecomposable finite dimensional modules over a tame hereditary algebra, in particular

- the partition of indecomposable finite dimensional modules into the three classes of preprojective, preinjective and regular modules, respectively;
- the category structure of each these three classes;
- the properties of the preprojective algebra(s) $\Pi = \Pi(H)$ and $\Pi' = \Pi'(H)$ attached to the preprojective component (resp. a distinguished τ^{-}-orbit of preprojective modules).

As a rule, these features will in particular be treated for the case of the Kronecker quiver $\circ \rightrightarrows \circ$.

I will stress the importance of tame hereditary algebras for many classification problems in representation theory, mention the link to weighted projective lines and to Iyama's theory of higher preprojective algebras, a key issue in his study of higher Auslander-Reiten theories.