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Exceptional sequences

Hagen Meltzer
(Szczecin University, Szczecin, Poland)

Abstract

Let k be a field and $\mathcal{H}$ be a hereditary k-category. An object E in $\mathcal{H}$ is called exceptional if $\mathrm{Ext}^1(E, E) = 0$ and End(E) is a skew field. A sequence $\epsilon = (E_1, \ldots, E_n)$ is called an exceptional sequence if $\mathrm{Hom}(E_i, E_j) = 0$ and $\mathrm{Ext}^1(E_i, E_j) = 0$ for $i > j$. If n coincides with the rank of the Grothendieck group of $\mathcal{H}$ then ϵ is called a full exceptional sequence. In good cases the objects of a full exceptional sequence can be ordered to form a tilting object in $\mathcal{H}$.

Crawley-Boevey has shown that the braid group on n stirngs acts transitively on the set of all exceptional sequences in the category of finite-dimensional modules over a hereditary k-algebra. Ringel generalized this result to the case of an artin algebra. The same result is true if $\mathcal{H}$ is the category of coheernt sheaves on a weighted projective line in the sense of Geigle and Lenzing. As a consequence we obtain that the endomorphism ring of any exceptional object is k.