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Liaison Theory

Elisa Gorla

University of Neuchatel

Switzerland

In this lecture, I plan to introduce liaison theory, and discuss plenty of examples. The theory of liaison or linkage formally started in the seventies, although it had been used before, in an ad-hoc manner. Roughly speaking, liaison aims at understanding the class of homogeneous ideals in the polynomial ring by partitioning it into families of ideals (the liaison classes) that can all be ultimately “linked” to the same ideal. In the special case of ideals which have no common associated primes, a linkage step consists of taking the intersection of the ideal that we wish study with another one, so that the intersection belongs to a well-studied family of ideals (complete intersections or Gorenstein ideals). The hope is that the ideal that we study can be linked to one that we understand better, and their intersection is simpler than each of the two ideals. Liaison also has an interesting “computational” aspect, in the sense that it sometimes allows us to compute the invariants of an ideal (such as its height, multiplicity, Hilbert function, and related numerical invariants), or to produce examples of ideals with given invariants.