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Some Geometric Applications of Macaulay's Inverse System

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Let $\mathbb{K}$ be field and let $S = \mathbb{K}[x_0, \ldots, x_n]$ be the ring of polynomials in $(n+1)$-variables. Let $R = \mathbb{K}[\partial/\partial x_0, \ldots, \partial/\partial x_n]$ be the ring of polynomials in partial differential operators.Then by defining a suitable scalar multiplication, S can be considered as an R-module. If I is a homogeneous ideal of R then the inverse system of I, which is denoted by I^{-1}, is the R-submodule of S which annihilated by every element of I.
In the years 1920s, I. S. Macaulay [3], invented the notion of inverse system of an ideal in the polynomial rings to study deeper properties of its ideals. Later on, in the final years of 20th century, A. Irrabino [2] used Macaulay's inverse system to study zero dimensional subschemes of projective n-space in order to determine their Hilbert functions. We give some algebro-geometric problems, which their solutions reduces to a problem about the Hilbert function of zero dimensional schemes and describe Irrabino's approach in determining this function. As another application, we explain how the inverse system can be used to answer a classical question which asks the conditions where for a fixed positive integers s, j, a homogeneous polynomial of degree $j$in $\mathbb{K}[x_0, \ldots, x_n]$ can be written as sum of homogeneous polynomials of degree j in general linear forms in $x_0, \ldots, x_n$.

References

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[3] F. S. Macaulay, *The Algebraic Theory of Modular Systems*, Cambridge University Press, 1994.