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## **Elimination Techniques for Parametric Polynomial Systems**

**Deepak Kapur**

*University of New Mexico*

*USA*

We will give a series of lectures on elimination of parametric polynomial systems. In the first part, we will discuss multivariate resultants for simultaneously eliminating many variables from a polynomial systems consisting of  $n + 1$  polynomials in  $n$  variables and  $m$  parameters. A special emphasis will be on the generalized Dixon resultant formulation introduced by Kapur, Yang and Saxena in an ISSAC 1994 paper [1]. The talk will give an overview of the results appearing in the Ph.D. dissertations of two of my former Ph.D. students Dr. Tushar Saxena [2, 5, 4, 3] and Dr. Arthur ChtCherba [6, 9, 8, 7]. A particular focus will be on identifying conditions under which exact resultants can be computed using the Dixon formulation. In cases when this is not possible, some known results about the degree and nature of the extraneous factors will be presented. Some open problem in this topic will also be discussed.

The second part of the talks will be on comprehensive Gröbner systems and comprehensive Gröbner basis of parametric polynomial systems [13, 14] (they were called parametric Gröbner systems and parametric Gröbner basis by me in my 1996 paper [10]). We will discuss recent algorithms developed in collaboration with Profs. Sun and Wang of the Chinese Academy of Sciences [11, 12]. Some open problems in this topic will also be discussed.

### **References**

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