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Computational Algebraic Geometry and Singularity Theory

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Many real life problems can be modeled by partial differential equations. Steady state of these equations at certain situation may be singular and represent complicated dynamics. Therefore the dynamic study of such problems is important. The steady state solutions can be reduced to

$$g(x, \lambda) = 0, \quad g(0, 0) = g_x(0, 0) = 0 \qquad (1)$$

by a technique called Liapunov-Schmidt reduction. We call equation (1) a bifurcation problem while g is called a singularity.

In this talk we show how we may use computational algebraic geometry tools for an automatic study of such bifurcation problems. We apply an equivalence relation called contact equivalence on the space of these functions such that every two functions from the same equivalence class have similar dynamical behaviour. From each class we choose the simplest representative call it nor- mal form and try to analyze it to attain information about the behaviour of the other functions in the corresponding class.

Here we introduce a new approach which can automatically obtain the results using computational algebraic geometry techniques. Because of the local structure of bifurcation problems we have to use the computations in local rings instead of common polynomial ring. Hence the monomial ordering, Gröbner basis, division algorithm and buchberger algorithm are substituted by the similar concepts in local rings. We have develop a maple package for an automatic bifurcation analysis of singularities. This is a joint project with Majid Gazor and Benyamin M.-Alizadeh.