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The Rosenfeld-Gröbner Algorithm

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The study of systems of differential and partial differential equations has a long history. The first steps were strongly motivated by physical problems for which the systems are most of the time *a priori* in closed form or could be reduced to it by linear algebra (using the Langrangian equations).

However, practical or theoretical considerations leaded soon to more complicated normal form computations issues and to the study of singular components, the meaning of which could be questioned.

In some posthumous paper [3,4], Jacobi already considers as well known that one may get a normal form by a suitable heuristic sequence of differentiations and eliminations but gives no general algorithm.

The work of Ritt [6], inspired by Riquier and Janet for PDEs, gave a strong basis to the study of differential equations from an algebraic standpoint, providing precise definitions for singular components and algorithmic tools to compute characteristic sets, that may be considered as some kind of normal forms. His algorithm suffers a great limitation, *viz.* the necessity of performing factorizations.

Boulier achieved in 1994 the first version of the RosenfeldGrbner algorithm which avoids this drawback. We will provide a precise description of the algorithm [1], which relies on two main tools: "Lazards lemma" and "Rosenfelds lemma" [7]. The last lemma means that a coherent system with regularity hypotheses, that may be reduced to the non vanishing of its separants, admits differential solutions iff it admits algebraic solutions. Lazards lemma implies that the associated differential ideal will be a radical ideal.

We will also provide theoretical links with the posthumous works of Jacobi, the geometric approach of Malgrange [5], as well as heuristical considerations that may improve the RosenfeldGrbner algorithm, such as assuming the dimensional conjecture or using Jacobis bound for a better choice of the ordering. We will conclude with some brief evocation of alternative tools for algebraic computations that can also be used to solve differential equations [2].

References

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