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On Cellular Resolution of Monomial Ideals

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Let $I \subset R = \mathbf{k}[y_1, ..., y_n]$ be a monomial ideal in the polynomial ring over a field $\mathbf{k}$ and let $G(I)$ be the unique minimal monomial generating set of I. Let X be a regular cell complex with $G(I)$ as its vertices. Let ϵ_X be an incidence function on X. Any face of X will be labeled by $\mathbf{m}_F$, the least common multiple of the monomials in $G(I)$ which correspond to the vertices of F. If $\mathbf{m}_F = y_1^{a_1}...y_n^{a_n}$, then the *degree* $\mathbf{a}_F$ is defined to be the exponent vector $e(\mathbf{m}_F) = (a_1, ..., a_n)$. Let RF be the free R-module with one generator in degree $\mathbf{a}_F$. The *cellular complex* $\mathbf{F}_X$ is the $\mathbb{Z}^n$-graded R-module $\bigoplus_{\emptyset \neq F \in X} RF$ with differentials

$$\partial(F) = \sum_{\emptyset \neq F' \in X} \epsilon(F, F') \frac{\mathbf{m}_F}{\mathbf{m}_{F'}} F'$$

If the complex $\mathbf{F}_X$ is exact, then $\mathbf{F}_X$ is called a cellular resolution of I. Alternatively, we say that I has a cellular resolution supported on the labeled cell complex X. If X is a polytope or a simplicial complex, then $\mathbf{F}_X$ is called *polytopal*, and *simplicial*, respectively.

The idea to describe a resolution of a monomial ideal by means of combinatorial chain complexes was initiated by Bayer, Peeva and Sturmfels [1], and was extended by Bayer and Sturmfels [2], and further extension was made by Jöllembeck and Welker [6]. Further contributions were given by Sinefakoupols [8], Mermin [7], Dochtermann and Engström [3],[4] and Goodarzi [5].

We consider monomial ideals that are facet ideals of a linear matroid of a set of vectors on which the only linear dependence is proportionality, i.e., some vectors could be scaler multiples of others. We prove that any such matroidal ideal has a linear resolution supported on a polytopal cell complex.

References

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