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Steps on Classification of Monomial Ideals with Linear Resolution

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Monomial ideals play an important role in studying the connections between commutative algebra and combinatorics. In fact, many problems in combinatorics can be encoded into monomial ideals, which allow us to use powerful methods in commutative algebra to solve the original question. Let $S = K[x_1, \dots, x_n]$ be the polynomial ring over a field K and I a homogeneous ideal of S. We say that I has d-linear resolution if I is generated by elements of degree d and there is a minimal free resolution for I such that $\beta_{i,i+j} = 0$ for all $j \neq d$, that is, the graded minimal free resolution of I is of the form

$$0 \longrightarrow S^{\beta_s}(-d-s) \longrightarrow \cdots \longrightarrow S^{\beta_1}(-d-1) \longrightarrow S^{\beta_0}(-d) \longrightarrow I \longrightarrow 0.$$

Proving that a class of ideals has d-linear resolutions is difficult in general.

The problem of existing 2-linear resolution is completely solved by R. Fröberg. Ideals of S generated by square-free monomials of degree 2 are corresponding to graphs and vice versa, any finite simple graph G is corresponding to a square-free monomial ideal named edge ideal of G. Fröberg proved that the edge ideal of a finite simple graph G has linear resolution if and only if the complementary graph $\bar{G}$ of G is chordal.

Another combinatorial objects corresponding to square-free monomial ideals are clutters which are special cases of hypergraphs. Let $[n] = \{1, \dots, n\}$. A clutter C on a vertex set $[n]$ is a set of subsets of $[n]$ (called circuits of C) such that if e_1 and e_2 are distinct circuits, then $e_1 \nsubseteq e_2$. To a clutter C its circuit ideal which is a square-free monomial ideal is corresponded. We say a d-uniform clutter C has linear resolution if the circuit ideal of the complimentary clutter $\bar{C}$ has a d-linear resolution.

In this talk, an overview of recent results on the partial classifications of uniform clutters with linear resolution will be presented.