The 12th Seminar on
Commutative Algebra and Related Topics, November 11 and 12, 2015
School of Mathematics, IPM, Tehran

Covering Techniques in Representation Theory

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Covering techniques in representation theory have become important after the work of Bongartz-Gabriel [BG], Gabriel [G] and Riedtmann [Ri]. In fact, at first Riedtmann [Ri] introduce coverings of the Auslander-Reiten quiver Γ_Λ of a representation-finite algebra Λ. Bongartz and Gabriel [BG] developed this notion to provide concrete algorithms which enable us to construct the Auslander-Reiten quivers for plenty of algebras.

Let $\Bbbk$ be a field and G be a group. In [G] Gabriel introduced the notion of Galois covering of locally bounded $\Bbbk$-categories with a G-action, to present a technique for the computation of the indecomposable modules over a representation-finite algebra.

Locally bounded G-categories have been well investigated in connection with a so-called covering technique in representation theory of algebras, see [G]. The orbit category $\mathcal{C}/G$ and the canonical functor $P : \mathcal{C} \longrightarrow \mathcal{C}/G$ are naturally constructed from these data, and one studies relationships between Mod-$\mathcal{C}$ and Mod-$(\mathcal{C}/G)$.

One of the most important results in this theory is the following theorem which is proved by Gabriel [G] and then completed by Martinez and De le Peña [MD]:

Theorem. let $\mathcal{C}$ be a locally bounded $\Bbbk$-category over an algebraically closed field $\Bbbk$ and let a group G act freely on $\mathcal{C}$. Then $\mathcal{C}$ is locally representation-finite if and only if $\mathcal{C}/G$ is so. In this case the push-down functor $P_\lambda : \text{Mod-}\mathcal{C} \longrightarrow \text{Mod-}(\mathcal{C}/G)$ associated with the Galois covering $P : \mathcal{C} \longrightarrow \mathcal{C}/G$ induces a bijection between the G-orbits of isomorphism classes of finitely generated indecomposable $\mathcal{C}$-modules and the isomorphism classes of finitely generated indecomposable $\mathcal{C}/G$-modules.

Asashiba in [As] generalized the covering technique for an arbitrary $\Bbbk$-categories with a G-action to apply covering techniques to usual additive categories such as the homotopy category $\mathbb{K}(\text{Prj-}\mathcal{C})$ of projectives. Using this generalization, we plan to extend the above theorem for algebras of finite Cohen-Macaulay type.

This talk is based on a joint work with H. Asashiba and R. Hafezi.

References

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