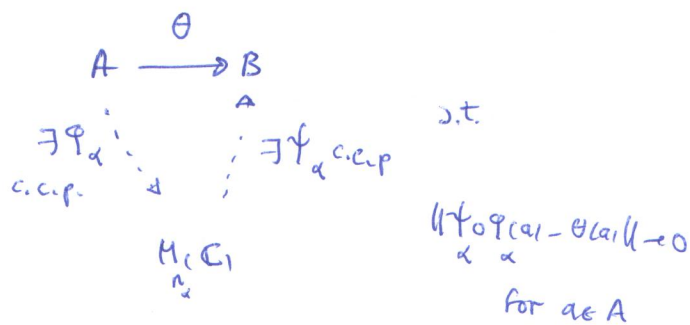


Margzele

linear map  $\theta: A \rightarrow B$  is called nuclear if



and  $C^*$ -alg  $A$  is called nuclear if

if  $\text{id}: A \rightarrow A$  is nuclear.

Example,  $C(X)$  for  $X = T_2 + \text{cpt.}$

Remarks:

i)  $\theta: A \rightarrow B$  nuclear iff for any  $\epsilon > 0$  &  $F \subseteq A$  finite

$\exists \varphi$  and  $\psi$  c.c.p map s.t.

$$\| \sum_{\alpha} \varphi_\alpha(a) - \theta(a) \| < \epsilon \text{ for } a \in F.$$

ii)  $\theta: A \rightarrow B$  nuclear iff  $\exists \varphi_\alpha: A \rightarrow C_n^\alpha$  c.c.p,  $\psi: C_n^\alpha \rightarrow B$  c.c.p

in which  $C_n^\alpha$  is f.d  $C^*$ -alg s.t.

$$\| \sum_{\alpha} \varphi_\alpha(a) - \theta(a) \| \rightarrow 0 \text{ for } a \in A.$$

pf of example: Using Partition of unity

and 1<sup>st</sup> part of remark

$F \subseteq A, \epsilon > 0 \exists \{U_i, \dots, U_n\} = \text{open cover finite}$

of  $X$  s.t.  $\| f_{U_i} - f_{U_j} \| < \epsilon$

for all  $x, y \in U_i$  &  $f \in F$

Consider  $\{\tau_i\}$  as a partition of unity of  $\{U_i\}$ .

$$\varphi: A \rightarrow \mathbb{C}^n$$

$$\varphi(f) = (f(x_1), \dots, f(x_n))$$

$$\psi: \mathbb{C}^n \rightarrow A$$

$$(d_1, \dots, d_n) \mapsto \sum_1^n d_i \tau_i$$

Then

$$\| \sum_{\alpha} \varphi_\alpha(f) - \theta(f) \| < \epsilon \text{ for } f \in F.$$

Cor. Every abelian VN-alg

is semi-discrete.

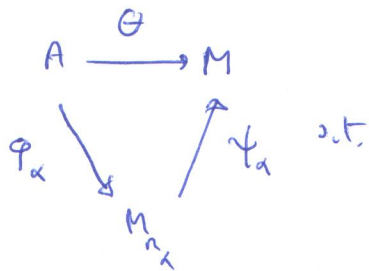
Aim:

Type I VN-alg are nuclear.

Remark:

Let  $\theta: A \xrightarrow{\text{linear}} M$ . Then  $\theta$  is

called weakly nuclear if



$$\langle \xi, \psi_\alpha \circ \varphi_\alpha(a) \rangle \rightarrow \langle \xi, \theta(a) \rangle \quad \xi \in M^*$$

- $A$  is nuclear iff  $A^{**}$  is semidiscrete.
- $A$  is type I  $C^*$ -alg iff  $A^{**}$  is type I  $W^*$ -alg.

$$\text{Type I } W^*\text{-alg} := \prod (A_i \otimes B(\mathcal{H}_i))$$

$\downarrow$   
 abelian  $W^*$ -alg

Lemma:  $M$   $W^*$ -alg acting on  $\mathcal{B}(\text{span}(P_\lambda))$   
 be a net of Proj s.t.  $P_\lambda \rightarrow 1$   
 on  $\text{dot-top}$  in  $B(\mathcal{H})$ . if  $P_\lambda M P_\lambda$   
 is semi-discrete then  $M$  is semi-discrete.

pf/  $\exists \lambda$  s.t.  $\|P_\lambda \xi - \xi\| < \varepsilon \quad \forall \xi \in \mathcal{H} \subseteq \mathcal{H}$   
 finite

$$\exists \tilde{\varphi}: P_\lambda M P_\lambda \rightarrow M_n(\mathbb{C}) \text{ and } \tilde{\psi}: M_n(\mathbb{C}) \rightarrow P_\lambda M P_\lambda \text{ s.t.}$$

$$\| \langle \psi \circ \varphi(P_\lambda M P_\lambda) \xi, \eta \rangle - \langle P_\lambda M P_\lambda \xi, \eta \rangle \| < \varepsilon$$

$$m \in \mathcal{F} \subseteq M \text{ finite} \\
 \varphi := \varphi(P_\lambda M P_\lambda)$$

Thm.  $M = \text{Type I } W^*\text{-alg} \Rightarrow M = \text{Semi-dis.}$

pf/ we construct the above Proj  
 mentioning in previous lemma:

$$\text{Let } \Omega \subseteq \mathcal{H} \text{ finite and } Q_\Omega: \mathcal{H} \xrightarrow{\text{Proj}} [\Omega] \subseteq \mathcal{H}$$

$$P_\Omega := 1 \otimes Q_\Omega \xrightarrow{\text{Sot.}} 1$$

$$P_\Omega (A \otimes B(\mathcal{H})) P_\Omega = A \otimes M_n(\mathbb{C})$$

Semi-discrete

A nuclear  $C^*$ -alg may have a  
 non-nuclear  $C^*$ -subalg but:

Thm: A sep.  $C^*$ -alg is type I  
 iff every its  $C^*$ -subalg is  
 nuclear.

$$\text{Example: } C_r^*(\mathbb{F}_r) \neq \text{type I } C^*\text{-alg} \\
 \text{but is nuclear}$$