The 14th Seminar on
Commutative Algebra and Related Topics, January 3 and 4, 2018
School of Mathematics, IPM, Tehran

Injective Dimension of Holonomic $\mathcal{D}$-modules and $\mathcal{F}$-finite Modules

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Let R be a commutative Noetherian ring with unit. If M is an R-module and $\mathrm{I} \subset R$ is an ideal, we denote the i-th local cohomology of M with support in I by $H^i_{\mathrm{I}}(M)$.

In a remarkable paper, [2], Lyubeznik used $\mathscr{D}$-modules to prove if R is any regular ring containing a field of characteristic 0 and I is an ideal of R, then

a) $H^i_{\mathfrak{m}}(H^i_{\mathrm{I}}(R))$ is injective for every maximal ideal $\mathfrak{m}$ of R.
b) $\operatorname{inj.dim}_R(H^i_{\mathrm{I}}(R)) \leq \dim_R(H^i_{\mathrm{I}}(R))$.

Later Lyubeznik [3] developed the theory of $\mathscr{F}$-modules over regular ring of char $p > 0$ and proved the same results in char $p > 0$.

By Lyubeznik results, the injective dimension of $H^i_{\mathrm{I}}(R)$ is bounded by its dimension.

Motivated by these results, I, [1, Theorem 4.1], proved that if $(R, \mathfrak{m})$ is a regular local ring which contains a field of characteristic $p > 0$ and M is an $\mathscr{F}$-finite module. Then $\dim_R M - 1 \leq \operatorname{inj.dim}_R M$. Also by using holonomic $\mathscr{D}$-modules, it is shown that , [1, Theorem 4.1], if $(R, \mathfrak{m})$ is a regular local ring which contains a field of characteristic 0 and $M = H^i_{\mathrm{I}}(R)_f$ for some $f \in R$. Then $\dim_R M - 1 \leq \operatorname{inj.dim}_R M$.

References

[1] M. Dorreh,*On the injective dimension of $\mathscr{F}$-finite modules and holonomic $\mathscr{D}$-modules*, Illinos Journal of math, to appear.
[2] G. Lyubeznik, *Finiteness properties of local cohomology modules(an application of D-modules to commutative algebra)*, Invent. Math.**113**(1993),41-55.
[3] G. Lyubeznik, *F-modules : applications to local cohomolgy and D-modules in chracteristic $P > 0$*, J.Reine Angew. Math.**491**(1997),65-130.