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Upper Bound for the Degree of the Inverse of a Birational Map

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This is a classical fact that if a plane Cremona map $F : P^2 \longrightarrow P^2$ is given by Polynomials of degree d, then so does its inverse. A generalization of this fact for birational maps from P^n to P^n is due to O. Gabber who showed in 80's that the inverse map has degree at most d^{n-1}. However if one considers an arbitrary projective variety X of P^n, then bounding the degree of the inverse map becomes a subtle challenge. This is the leitmotiv of this talk which answers a question by Jeremy Blanc. We start from Plane Cremona maps to show how the Betti table of the base ideal constrains birationality. Moving from Plane case to higher dimensions, we encounter the role of higher polynomial equations of the base ideal, consequently the structure of the Rees algebra appears. A new Criterion for Birationality is presented which provides information on the inverse map for any birational map $F : X \longrightarrow Y$ of projective varieties. An emphasis will be on the particular cases where the graph of the Birational map is arithmetically Cohen-Macaulay.

This talk is based on some joint works with A. Simis