The 14th Seminar on
Commutative Algebra and Related Topics, January 3 and 4, 2018
School of Mathematics, IPM, Tehran

The Ideal of Maximal Flags of a Poset

Amin Nematbakhsh
IPM

Iran

Let P be a poset. A *flag* in P is a chain of elements $p_1 < \cdots < p_n$ in P. Let $R = \Bbbk[x_P]$ be a polynomial ring in indeterminates x_i, for all $i \in P$. For a subset $I = \{p_1, \ldots, p_n\}$, we denote the squarefree monomial $\prod_{i \in I} x_i$ by x_I. We define the *flag ideal* $\mathcal{F}(P)$ of P to be the squarefree monomial ideal generated by all monomials x_I where I is a maximal chain of elements in P.

In [3], Hibi and Herzog gave a characterization of bipartite graphs for which their edge ideals is Cohen-Macaulay. Inspired by their work, Villarreal gave a characterization of unmixed edge ideals of bipartite graphs in [5]. Furthermore, in [1], Corso and Nagel show that the edge ideal of a bipartite graph has a linear resolution if and only if it is a Ferrers graph.

The edge ideal of a bipartite graph is a special case of flag ideals (a bipartite graph can be viewed as the Hasse diagram of a rank 2 poset). There exist some attempts to generalize the above-mentioned results to edge ideals of uniform multipartite clutters, but we believe the correct setting which generalizes the results on edge ideals of bipartite graph to monomial ideals of higher degree is by the class of flag ideals.

In this talk, we show that the above-mentioned results for edge ideals of bipartite graphs also hold for flag ideals $\mathcal{F}(P)$, where P is a graded poset. More precisely, we give characterizations for unmixed and Cohen-Macaulay flag ideals. We also show that a flag ideal has a linear resolution if and only if the bipartite graphs on elements of successive rank in the poset, are all Ferrers graphs. In addition, we provide structural results on the mutigraded Betti numbers of such flag ideals, generalizing the similar results in [2] for letterplace ideals. The results can be found in [4].

References

[1] Alberto Corso and Uwe Nagel. Monomial and toric ideals associated to Ferrers graphs. *Transactions of the American Mathematical Society*, 361(3):1371–1395, 2009.

[2] Alessio D'Alì, Gunnar Fløystad, and Amin Nematbakhsh. Resolutions of letterplace ideals of posets. *Journal of Algebraic Combinatorics*, 45(3):911–930, 2017.

[3] Jürgen Herzog and Takayuki Hibi. Distributive lattices, bipartite graphs and alexander duality. *Journal of Algebraic Combinatorics*, 22(3):289–302, 2005.

[4] Amin Nematbakhsh. The ideal of maximal flags of a poset. *preprint, arXiv: 1706.05742*, 2017.

[5] Rafael H. Villarreal. Unmixed bipartite graphs. *Revista Colombiana de Matemticas*, 41(2):393–395, 2007.