Generic artinian quotients of $k[x,y,z]$.

$(Q, \mathfrak{m}, k)$ regular local ring

$I \subseteq Q$ a perfect ideal

$F.$ min'l free resolution of $R = Q/I$ over Q

$\operatorname{rank} F_1 = m$

$\operatorname{rank} F_3 = n$

$A = H(F. \otimes_Q k) = \operatorname{Tor}^Q_*(R, k)$

$\operatorname{pd}_Q R = \operatorname{codepth} R$

$= \operatorname{codim} R$

Recall:

$\operatorname{codim} R = 1 \Rightarrow R$ hypersurface codim $R = 2 \Rightarrow$

(i) R c.i. $\iff$ (i) $A = \Lambda(k^2)$

(ii) R Golod $\iff$ (ii) $(A_{\geq 1})^2 = 0$

codim $R = 3 \Rightarrow R$ is

(i) c.i $\iff$ $C(3)$

(ii) Golod $\iff$ $H(0,0)$

(iii) Gor. not c.i. $\iff$ $G(r=m)$

(iv) sth. else $\iff$ $G(r<m)$

$H(p,q)$ $p+q>0$

B, T

Thm [14] If codim $R = 3$ then I is linked to a grade 3 perfect ideal J s/t Q/J is c.i. or Golod

<u>Recall</u> If codim $R = 2$ then I is linked to a grade 2 perfect ideal J s/t Q/J is c.i.

— x —

The realization problem [3]:

- conjectures for I of grade 3.
- If J has grade 3 and $\dim Q/J - \operatorname{depth} Q/J = 2$ then J is of class $H(0,0)$
- If J has grade 3 and $\dim Q/J - \operatorname{depth} Q/J = 1$, "we know nothing!"

Software for experiments:

- [7] Finds the products on a graded free resolution of length 3 over $\dfrac{k[x_1,\ldots,x_n]}{\mathfrak{a}}$

- [9] Classifies quotients R of codepth 3 of $k[x_1,\ldots,x_n]$.

$$P_R(t) = \frac{(1+t)^2}{g(t,m,n,p,q)}$$

$$I_R(t) = \frac{f(t,m,n,p,q,r)}{g(t,m,n,p,q)}$$

Classification Alg. [10].

Specializing:

$Q = k[x,y,z]$, $\mathfrak{m} = (x,y,z)$

$\operatorname{codim} R = 3 \iff R$ artinian

$\iff \mathfrak{m}^{s+1} \subseteq I$

I homogeneous

$s :=$ socle degree of R.

$$
\begin{aligned}
A &= \operatorname{Tor}^Q_*(R,k) \\
&= H(F_\cdot \otimes_Q k) \\
&= H(F_\cdot \otimes_Q K^Q(x,y,z)) \\
&= H(R \otimes_R K^Q(x,y,z)) \\
&= H(K^R(x,y,z))
\end{aligned}
$$

<< 1988 :

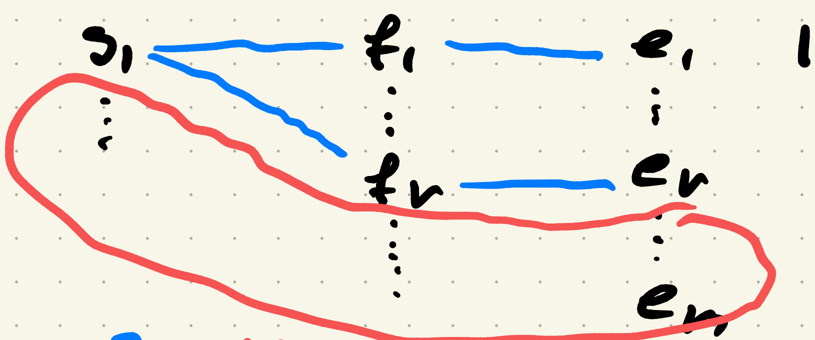
R is Gorenstein $\iff$ $A = H(K^R(x,y,z))$ is a Poincaré duality algebra

R Golod $\implies$ $\left(H(K^R(x,y,z))_{\geq 1}\right)^2 = 0$

<u>Focus $n = 2$:</u>

Generically these rings are of class "G(r)" with $0 \leq r \leq n-3$

Take a look at A for $G(r)$



$A = P \ltimes V$

$\mathrm{Soc}\, R = k\langle u, v\rangle$

$|v| = s$

$|u| = s_1$ $\qquad s_1 \leq s$

If $s_1 \approx s$, then R Golod

If $s_1 << s$, then R is of class $G(r)$ $(r > 0)$

"Proof of behavior"

$I = I_1 \cap I_2$: Q/I_1 and Q/I_2 Gor.

$$\begin{tikzcd}
 & F_\cdot^{r} \arrow[r, "\text{mod.}"'] \arrow[d] & F_\bullet \arrow[r, "\text{alg.}"'] \arrow[d] & F'_\bullet \arrow[d] & \\
0 \arrow[r] & K \arrow[r] & R \arrow[r] & Q/I_2 \arrow[r] & 0
\end{tikzcd}$$