Abstracts of the Seminar Talks

(In Alphabetical Order)

Invited Speakers

(In Alphabetical Order)

The 18th (Virtual) Seminar on
Commutative Algebra and Related Topics,
January 9 - 11, 2022, School of Mathematics, IPM, Tehran

Free Resolutions Over Algebras Defined by Spaces of Quadrics of Low Codimension

Rasoul Ahangari Maleki
IPM, Iran

Let $\mathbf{k}$ be an algebraically closed field and R be a standard graded quadratic $\mathbf{k}$-algebra with $\dim_{\mathbf{k}} R_2 \leq 3$. In this lecture we will talk about free resolutions of graded modules over R. More precisely, we show that there is a graded surjective Golod homomorphism $\varphi : P \to R$ such that P is a complete intersection. The existence of such a map implies that every finitely generated graded R-module has a rational Poincaré series. Moreover, we show that R is absolutely Koszul if and only if R is Koszul if and only if R is not a trivial fiber extension of a standard graded $\mathbf{k}$-algebra with Hilbert series $(1 + 2t - 2t^3)(1 - t)^{-1}$.
This talk is based on a joint work with Liana Şega.

*The 18th (Virtual) Seminar on
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Extendably Shellable Simplicial Complexes from the Algebraic Point of View

Mina Bigdeli
IPM, Iran

One of the large classes of Cohen–Macaulay polynomial rings arises from the ideals attached to some combinatorial objects called *pure shellable* simplicial complexes. The substantial role of Cohen–Macaulay rings at the crossroads of Commutative Algebra, Combinatorics, and Topology motivates us to analyze the structure of shellable simplicial complexes and study their algebraic behavior through Stanley–Reisner theory.

The question that we will focus on is that"given a pure shellable simplicial complex which is not the skeleton of a simplex, whether one gets stuck in the process of extending it to a larger shellable complex". The negative answer to this question has been conjectured by R. S. Simon in 1994, which attracted the attention of some researchers in Combinatorics, Topology, as well as Algebra.

In this talk, we will state the algebraic interpretation of this conjecture and explain how the employment of some algebraic tools helps with attacking it. We will then review some results in this direction.

This talk is mainly based on a joint work with A. A. Yazdan Pour and R. Zaare-Nahandi.

References

[1] R.S. Simon, *Combinatorial properties of cleanness*, J. Algebra, **167**, 361–388 (1994).

Minimal Resolutions and Rigidity of Ext and Tor

Lars Winther Christensen
Texas Tech University, USA

Homological dimensions of modules and complexes can be characterized in terms of vanishing of derived functors: Ext and Tor. For finitely generated modules it is classic that vanishing can be detected with cyclic modules, in the local case even with the simple modules. I will survey results from the last 5 years that deal with the non-finitely generated case.

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Generalization of the Auslander-Reiten Duality for Algebras

Ali Mahinfallah
Alzahra University and IPM, Iran

Let R be a Cohen-Macaulay local ring of dimension d with canonical module ω_R. In this talk, we extend the Auslander-Reiten duality theorem to certain R-algebras. As a consequence of our argument, we obtain a criterion for projective modules over certain R-algebras in terms of the vanishing of Ext modules.

The 18th (Virtual) Seminar on
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On Gerko's Strongly Tor-independent Modules

Keri Sather-Wagstaff
Clemson University, USA

Gerko proves that if an artinian local ring (R,m) possesses a sequence of strongly Tor-independent modules of length n, then $m^n \neq 0$. This generalizes readily to Cohen-Macaulay rings. We present a version of this result for non-Cohen-Macaulay rings.

This is joint work with Hannah Altmann. https://arxiv.org/abs/2012.03361

The 18th (Virtual) Seminar on
Commutative Algebra and Related Topics,
January 9 - 11, 2022, School of Mathematics, IPM, Tehran

Naive Liftings and Connections

Yuji Yoshino
Okayama University, Japan

Naive liftability of dg modules is the new concept introduced by M. Ono, S. Nasseh, and Y. Yoshino for the purpose of unifying the ideas of lifting and weak lifting for modules over commutative rings. In this talk, I will show how we get the obstruction class of naive liftings. Surprisingly, this class coincides with the Atiyah class that has been introduced by Buchweitz and Flenner.

Contributed Speakers

(In Alphabetical Order)

The 18th (Virtual) Seminar on
Commutative Algebra and Related Topics,
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Unexpected Behaviour of the Number of Generators of Powers of Monomial Ideals

Reza Abdolmaleki
IASBS, Zanjan, Iran

Let I be a graded ideal in the polynomial ring $S = K[x_1, \dots, x_n]$ and $\mu(I)$ denote the minimal number of generators of I. It is well known that the function $f(k) = \mu(I^k)$, for $k \gg 0$, is a polynomial in k. This fact implies that if I is not a principal ideal, then $\mu(I^k) < \mu(I^{k+1})$ for $k \gg 0$. Now, the question is: What is the behavior of $\mu(I^k)$ for small integers k? Is it true that $\mu(I^k) < \mu(I^{k+1})$ for all k when I is not a principal ideal?

In the case that I is an equigenerated graded ideal, the answer is positive [5]. But this question dose not have a positive answer in general (see [3], [4]). Here we give more surprising results:
For any given positive integer k_0, we construct a monomial ideal $I \subset K[x_1, x_2]$ such that the function $f(k) = \mu(I^k)$ has a local maximum (minimum) at $k = k_0$. As a more surprising result, for any given number q, we obtain an ideal of height 2 such that the function $f(k)$ has at least q local maxima ([1]). We also construct families of monomial ideals which, for any given number k_0, have the property that the number of generators of the powers is strictly decreasing up to the power k_0 ([2]).

Based on a joint work with J. Herzog and R. Zaare-Nahandi, and a joint work with S. Kumashiro.

References

[1] R. Abdolmaleki, J. Herzog, R. Zaare-Nahandi, *On the initial behaviour of the number of generators of powers of monomial ideals*, Bull. Math. Soc. Sci. Math. Roumanie , Tome **63** (111), No. 2, 2020, 119–129

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[3] S. Eliahou, J. Herzog, M. M. Saem, *Monomial ideals with tiny squares*, J. Algebra, **514** (2018), 99–112.

[4] O. Gasanova, *Monomial ideals with arbitrarily high tiny powers in any number of variables*, Communications in Algebra, 48:11, 4824–4831. DOI: 10.1080/00927872.2020.1772276

[5] J. Herzog, M. Mohammadi Saem, N. Zamani, *The number of generators of the powers of an ideal*, Internat. J. Algebra Comput., **29** (2019), 827–847.

*The 18th (Virtual) Seminar on
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Characterizing Certain Semidualizing Complexes via their Betti and Bass Numbers

Kosar Abolfath Beigi
Alzahra University, Iran

Christensen introduced the notion of semidualizing complexes. Two distinguished types of semidualizing complexes are the shifts of the underlying rings and dualizing complexes. These are precisely semidualizing complexes with finite projective dimension and semidualizing complexes with finite injective dimension. It is known that a homologically bounded complex X with finitely generated homology modules is a dualizing complex for a local ring R if and only if there exists an integer d such that the dth Bass number of X is 1 and the other Bass numbers of X are 0.

By the work of Bass, every Cohen-Macaulay local ring of type 1 is Gorenstein. Christensen et al. improved this to the statement that if R is a Cohen-Macaulay local ring and $\mu_R^n(R) = 1$ for some $n \geq 0$, then R is Gorenstein of dimension n. Vasconcelos conjectured that every local ring of type 1 is Gorenstein. Foxby solved this conjecture for equicharacteristic local rings. The general case was answered by Roberts. Let C be a semidualizing complex for a local ring R, and set $n := \sup C$ and $d := \dim_R C$. Suppose that $\mu_R^d(C) = 1$. Is it true that C is a dualizing complex for R? What about when $\beta_n^R(C) = 1$?

In this talk, I will discuss how Betti and Bass numbers of a semidualizing complex can be used to analyze it. Also, I will present some criteria for semidualizing complexes that are either the shifts of the underlying ring or dualizing complex with the aid of their Betti and Bass numbers.

This talk is based on a joint work with Kamran Divaani-Aazar and Massoud Tousi.

The 18th (Virtual) Seminar on
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January 9 - 11, 2022, School of Mathematics, IPM, Tehran

Homological Theory of Serre Quotient

Ramin Ebrahimi
University of Isfahan and IPM, Iran

Inspired by the work of C. Psaroudakis [5], for an abelian category $\mathcal{A}$ and a Serre subcategory $\mathcal{C}$, we investigate homological aspects of the quotient category $\frac{\mathcal{A}}{\mathcal{C}}$. The existence of section functor and injective objects are not assumed.

As an application we will provide a necessary and sufficient condition for a contravarintly finite subcategory $\mathcal{X}$ of an abelian category to be n-rigid (i.e. $\mathrm{Ext}^{1,\cdots,n-1}(\mathcal{X},\mathcal{X}) = 0$). This result in this form is due to S. Kvamme [4], and is implicitly used in [2].

References

[1] R. Ebrahimi, Homological theory of Serre quotient, preprint (2021), arXiv:2107.13623v1.
[2] R. Ebrahimi, A. Nasr-Isfahani, Higher Auslander's formula, *Int. Math. Res. Not. IMRN 0* (2021) 1–18.
[3] P. Gabriel, Des catégories abéliennes, *Bull. Soc. Math. France*, **90** (1962), 323–448.
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Affine Semigroups of Maximal Projective Dimension

Kriti Goel
Indian Institute of Technology Gandhinagar, India

A submonoid of $\mathbb{N}^d$ is of maximal projective dimension (MPD) if the associated affine semigroup ring has the maximum possible projective dimension. Such submonoids have a nontrivial set of pseudo-Frobenius elements. We generalize the notion of symmetric semigroups, pseudo-symmetric semigroups, and row-factorization matrices for pseudo-Frobenius elements of numerical semigroups to the case of MPD-semigroups in $\mathbb{N}^d$. We prove that under suitable conditions these semigroups satisfy the generalized Wilf's conjecture. We prove that the generic nature of the defining ideal of the associated semigroup ring of an MPD-semigroup implies uniqueness of row-factorization matrix for each pseudo-Frobenius element. Further, we give a description of pseudo-Frobenius elements and row-factorization matrices of gluing of MPD-semigroups. We prove that the defining ideal of gluing of MPD-semigroups is never generic.

Based on a joint work with Om Prakash Bhardwaj and Indranath Sengupta.

The 18th (Virtual) Seminar on
Commutative Algebra and Related Topics,
January 9 - 11, 2022, School of Mathematics, IPM, Tehran

Simplicial Affine Semigroups with Monomial Minimal Reduction Ideals

Raheleh Jafari
Kharazmi University and IPM, Iran

Let $S \subset \mathbb{N}^d$ be a simplicial affine semigroup with the extremal rays $\mathbf{a}_1, \ldots, \mathbf{a}_d$. The semigroup ring $R = \mathbb{K}[S]$ is of Krull dimension d with a unique monomial maximal ideal $\mathfrak{m}$. The study of affine semigroups and corresponding semigroup rings has many applications in different areas of mathematics; for instance, it gives the combinatorial background for the theory of toric varieties. Affine simplicial semigroups provide also a natural generalization of numerical semigroups, whose theory has been developed mainly in connection with the study of curve singularities.

In the case of numerical semigroup rings, a remarkable property is that the monomial maximal ideal always has a monomial minimal reduction, which is the principal ideal generated by the monomial of minimal positive degree. This is a key fact to explore the connection between the associated graded ring $\mathrm{gr}_{\mathfrak{m}}(R) = \oplus_{i=0}^{\infty} \mathfrak{m}^i/\mathfrak{m}^{i+1}$, the ring R, and the associated semigroup. However, in the case of affine simplicial semigroup rings, a monomial minimal reduction for $\mathfrak{m}$ in general does not exist.

We firstly observe that, if such a minimal reduction exists, it has to be generated by the monomials $\mathbf{x}^{\mathbf{a}_1}, \ldots, \mathbf{x}^{\mathbf{a}_d}$ corresponding to the extremal rays, provided that $\mathbb{K}$ is infinite. We characterize the existence of the monomial minimal reduction of $\mathfrak{m}$. In the case that $I = (\mathbf{x}^{\mathbf{a}_1}, \ldots, \mathbf{x}^{\mathbf{a}_d})$ is a minimal reduction for $\mathfrak{m}$, we study the Cohen-Macaulay and Gorenstein properties of the associated graded ring and provide several bounds for the reduction number with respect to I.

Finally we get a formula for the Castelnouvo-Mumford regularity of $\mathrm{gr}_{\mathfrak{m}}(R)$, when $\mathfrak{m}$ has monomial minimal reduction and $\mathrm{gr}_{\mathfrak{m}}(R)$ is Cohen-Macaulay.

References

[1] M. D'Anna, R. Jafari and F. Strazzanti, Simplicial affine semigroups with monomial minimal reduction ideals, to appear in Mediterranean Journal of Mathematics. An earlier preprint is available at https://arxiv.org/abs/2107.09970.

The 18th (Virtual) Seminar on
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Extending Tiling Subcategories to One-Point Extensions by Projectives

Somayeh Sadeghi
University of Isfahan, Iran

Let $\mathcal{A}$ be an abelian category. A cotorsion torsion triple is a triple of subcategories $(\mathcal{C}, \mathcal{T}, \mathcal{F})$ in $\mathcal{A}$ such that $(\mathcal{C}, \mathcal{T})$ is a cotorsion pair and $(\mathcal{T}, \mathcal{F})$ is a torsion pair. In [2], A. Beligiannis, and independently in [1], the authors proved that there exists a bijection between the collection of all cotorsion torsion triples and the collection of all tilting subcategories. In this talk, after reviewing the results of [1], we study the behavior of tilting subcategories and cotorsion torsion triples over one-point extensions by projective modules.

The talk is based on the joint work with Javad Asadollahi.

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*The 18th (Virtual) Seminar on
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Cohen-Macaulay Differential Graded Algebras and their Applications

Liran Shaul
Charles University, Czech Republic

The aim of this talk is to explain how to generalize the Cohen-Macaulay property to the setting of commutative differential graded rings. We will give many different equivalent characterizations of this notion and show that it arises naturally. In particular, we will explain that the derived quotient of a Cohen-Macaulay ring by any finite sequence of elements is a Cohen-Macaulay DG-ring. As applications, we will present a derived generalization of Hironaka's miracle flatness theorem, as well as a differential graded generalization of the Auslander-Buchsbaum-Serre theorem about localization of regular local rings.

The 18th (Virtual) Seminar on
Commutative Algebra and Related Topics,
January 9 - 11, 2022, School of Mathematics, IPM, Tehran

Almost Canonical Ideals and gAGL Rings

Francesco Strazzanti
University of Turin, Italy

Let $(R, \mathfrak{m})$ be a one-dimensional Cohen-Macaulay local ring. When R has minimal multiplicity, Goto, Matsuoka, and Phuong [4] proved that R is almost Gorenstein if and only if the algebra $\mathfrak{m} : \mathfrak{m}$ is Gorenstein. Afterwards, Chau, Goto, Kumashiro, and Matsuoka [1] introduced the notion of 2-almost Gorenstein local ring (briefly 2-AGL) and proved that if R has minimal multiplicity and $\mathfrak{m} : \mathfrak{m}$ is local, then R is 2-AGL if and only if $\mathfrak{m} : \mathfrak{m}$ is almost Gorenstein, but not Gorenstein, and its residue field is isomorphic to $R/\mathfrak{m}$.

I will explain some generalizations of these results, without assuming that R has minimal multiplicity or $\mathfrak{m} : \mathfrak{m}$ is local. With this aim, I will start by studying the particular case of numerical semigroup rings, and then I will introduce the notions of almost canonical ideal and gAGL ring.

References

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[2] M. D'Anna, F. Strazzanti, *Almost canonical ideals and* GAS *numerical semigroups*, Commun. Algebra **49** (2021), no. 8, 3534-3551.

[3] M. D'Anna, F. Strazzanti, *When is* $\mathfrak{m} : \mathfrak{m}$ *an almost Gorenstein ring?*, Rev. Mat. Complut., to appear.

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The 18th (Virtual) Seminar on
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Shedding Vertices and Ass-Decomposable Monomial Ideals

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The shedding vertices of simplicial complexes are studied from an algebraic point of view. Based on this perspective, we introduce the class of ass-decomposable monomial ideals which is a generalization of the class of Stanley-Reisner ideals of vertex decomposable simplicial complexes. The recursive structure of ass-decomposable monomial ideals allows us to find a simple formula for the depth, and in squarefree case, an upper bound for the regularity of such ideals.