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Toric and Toroidal Morphisms under the Affine Microscope

Razieh Ahmadian
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A ***toric variety*** is a normal variety $\mathfrak{X}$ that contains an *algebraic torus* T as a Zariski open subset such that the natural action of T on itself extends to an *algebraic action* of T on $\mathfrak{X}$, i,e., an action $T \times \mathfrak{X} \to \mathfrak{X}$ which is given by a morphism of algebraic varieties. In toric category, equivariant maps that respect the torus action, the so-called toric maps, are of interest. In more detail, if $\mathfrak{X}_1$ and $\mathfrak{X}_2$ are toric varieties, a morphism $\Phi : \mathfrak{X}_1 \to \mathfrak{X}_2$ of algebraic varieties is ***toric*** if it maps the torus $T_1 \subset \mathfrak{X}_1$ into $T_2 \subset \mathfrak{X}_2$, and $\Phi|_{T_1}$ is a group homomorphism. In particular, if we have affine toric varieties, there is a simple characterization of toric morphisms which helps us to give a careful description of a toric map between nonsingular affine toric varieties in terms of a system of monomial equations and its exponents matrix. As an immediate application, we can characterize when a morphism of nonsingular quasi-projective varieties are toroidal, i.e., it is locally analytically toric. In this talk, these results will be discussed after introducing some basic tools in toric geometry.

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Algebraic Invariants of Interval Subdivided Complexes

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Walker introduced the interval subdivision of a simplicial complex. This talk will explore the ring-theoretic properties of Stanley-Reisner rings associated with simplicial complexes undergoing interval subdivisions. The focus encompasses computational aspects such as dimension, Hilbert series, multiplicity, local cohomology, depth, and regularity of the resulting Stanley-Reisner ring. Moreover, I will present some interesting results concerning the non-vanishing of graded Betti numbers and bounds on Betti numbers for these Stanley-Reisner rings.

This is my joint work with Shaheen Nazir, which recently appeared in the Communications in Algebra.

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Algebraic Invariants of Codes on Weighted Projective Planes

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Weighted projective spaces are natural generalizations of classical projective spaces having rich structures and exhibiting interesting algebraic geometric properties. They have been regarded as convenient ambient spaces to create interesting classes of linear codes over finite fields in the literature (see, [1],[2],[3]). Weighted Projective ReedMuller codes were introduced by Sørensen in [3].

The purpose of this talk is to introduce these codes over a finite field, to give results for their main parameters, and to reveal the role of computer algebra packages to study some of the relevant combinatorial commutative algebraic invariants. We pay a particular attention on two dimensional case to obtain more explicit information about the minimal free resolution of the vanishing ideal of the weighted projective plane $\mathbb{P}(1, a, b)$ over $\mathbb{F}_q$. This yields to the Hilbert function giving the dimension of the code and regularity index which is crucial to eliminate trivial codes. We also compute the minimum distance of codes on the weighted projective space of the form $\mathbb{P}(1, 1, b)$ and in this talk we will share these results.

This is a joint work with Mesut Sahin.

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On the Epsilon Multiplicity of Monomial Ideals

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Suppose that I is an ideal in a Noetherian local ring $(R, \mathfrak{m})$ of Krull dimension d. Ulrich and Validashti define the ε-multiplicity of I to be

$$\varepsilon(I) := \limsup_{n\to\infty} \frac{\lambda_R\left(H^0_{\mathfrak{m}}\left(R/I^n\right)\right)}{n^d/d!}.$$

This invariant can be seen as a generalization of the classical Hilbert-Samuel multiplicity. Cutkosky shows that the '$\limsup$' in the definition of ε-multiplicity can be replaced by a limit if R is analytically unramified. Cutkosky-Hà-Srinivasan-Theodorescu give an example of a prime ideal in a regular local ring whose ε-multiplicity is irrational. Such pathological behaviour occurs because the saturated Rees algebra $\oplus_{n\geq 0}\left(I^n :_R \mathfrak{m}^{\infty}\right)$ can be non-Noetherian. A celebrated result due to Herzog-Hibi-Trung states that the saturated Rees algebra of a monomial ideal in a polynomial ring over a field is finitely generated. In particular, the ε-multiplicity of a monomial ideal is a rational number. In this talk, we shall discuss various generalizations of this result.

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Minimal Free Resolutions, Shellability, Matroids, and Codes

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Minimal free resolutions are classical objects in commutative and homological algebra, and their study goes back at least to Hilbert. We will be particularly interested in graded minimal free resolutions of polynomial algebras quotiented by monomial ideals, and in particular, Stanley-Reisner rings of abstract simplicial complexes. These have some nice properties when the corresponding simplicial complexes are shellable. An important class of shellable complexes is that of simplicial complexes formed by independent subsets of a matroid. This includes, as a special case, simplicial complexes arising from linear error correcting codes. A relatively recent work of Johnsen and Verdure shows that some of the fundamental parameters of a linear code are closely related to the Betti numbers arising from graded minimal free resolutions of the Stanley-Reisner rings of matroid complexes corresponding to these codes. However, computing Betti numbers is usually a hard problem. But it is tractable if the free resolution is "pure".

We will first outline these developments while making an attempt to keep the prerequisites at a minimum. We shall then describe an intrinsic characterization of purity of graded minimal free resolutions associated with linear codes. Further, we will discuss a characterization of (generalized) Reed-Muller codes that admit a pure resolution.

If there is time and interest, we will also discuss the case of rank metric codes and possible analogues of some of the notions and results discussed above.

The first part of this talk is based on joint works with Prasant Singh and also with Rati Ludhani. The second part is based mainly on a joint work with Rakhi Pratihar and Tovohery Randrianarisoa.

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Expecting the Unexpected: Quantifying the Persistence of Unexpected Hypersurfaces

Elena Guardo
University of Catania, Italy

Let X be a reduced subscheme in $\mathbb{P}^{\Bbbk}$. We say that X admits an unexpected hypersurface of degree d and multiplicity m if the imposition of having multiplicity m at a general point P fails to impose the expected number of conditions on the linear system of hypersurfaces of degree d containing X. We introduce new methods for studying unexpectedness, such as the use of generic initial ideals and partial elimination ideals to clarify when it can and when it cannot occur. We formulate a new way of quantifying unexpectedness (our AV sequence), which allows us detect the extent to which unexpectedness persists as increases but remains constant. We also study how knowledge of the Hilbert function, together with certain geometric assumptions, can provide information about unexpected hypersurfaces.

This is a joint work with G. Favacchio, B. Harbourne and J. Migliore

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The Skeleton of a Convex Polytope

Takayuki Hibi
Osaka University, Japan

Let $\mathrm{sk}(\mathcal{P})$ denote the 1-skeleton of a convex polytope $\mathcal{P}$. Let C be a clique (=complete subgraph) of $\mathrm{sk}(\mathcal{P})$ and $\mathrm{conv}(C)$ the convex hull of the vertices of $\mathcal{P}$ belonging to C. In general, $\mathrm{conv}(C)$ may *not* be a face of $\mathcal{P}$. It will be proved that $\mathrm{conv}(C)$ is a face of $\mathcal{P}$ if $\mathcal{P}$ is either the order polytope $\mathcal{O}(P)$ of a finite partially ordered set P or the stable set polytope $\mathrm{Stab}(G)$ of a finite simple graph G. In other words, when $\mathcal{P}$ is either $\mathcal{O}(P)$ or $\mathrm{Stab}(G)$, the simplicial complex consisting of simplices which are faces of $\mathcal{P}$ is the clique complex of $\mathrm{sk}(\mathcal{P})$.

This is a joint work with Aki Mori (arXiv:2307.08447).

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Some Algebraic and Combinatorial Properties of Graphs under Graph Operations

Fahimeh Khosh-Ahang Ghasr
Ilam University, Iran

Throughout this talk, the vertex decomposability and shellability of graphs formed from other graphs by various operations are investigated. Also among the other things, by using some graph operations, new classes of Cohen-Macaulay graphs from previous ones are presented.

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Polynomial Invariant Rings in Modular Invariant Theory

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A Let G be a finite group acting linearly on a finite-dimensional vector-space V over a field K. Let R denote the symmetric algebra on V^*; then G acts as graded K-algebra automorphisms on R. If R^G is a polynomial ring, then G is generated by elements that act as pseudo-reflections on V. The converse holds when $|G|$ is invertible in K. The above results are Shephard-Todd-Chevalley-Serre theorem. If $\mathrm{char}(K) = p > 0$ and G is a p-group, then a conjecture of Shank-Wehlau-Broer asserts that R^G is a polynomial ring if R^G is a direct summand of R as an R^G-module. We verify this conjecture in dimension three and prove some results supporting the conjecture in dimension four.

This is joint work with Mandira Mondal.

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On the Reduction of Hankel Determinantal Ideals

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Federal University of Rio de Janeiro, Brazil

The generic Hankel matrix is the "super"-symmetric matrix of the following form. This type of matrix is the main member of the family of 1-generic matrices

$$H = \begin{pmatrix} x_1 & x_2 & \dots & x_{m-1} & x_m \\ x_2 & x_3 & \dots & x_m & x_{m+1} \\ \vdots & \vdots & \dots & \vdots & \\ x_{m-1} & x_m & \dots & x_{2m-3} & x_{2m-2} \\ x_m & x_{m+1} & \dots & x_{2m-2} & x_{2m-1} \end{pmatrix}.$$

The main question that we address is: A minimal reduction of sub-maximal minors of H is the gradient ideal of the determinant. We also determine the reduction number associated to gradient ideal, which is m-2. We leave an open question regarding reduction ideals of degenerations of these ideals. By degeneration of the Hankel matrix we mean to set all last r variables zero, whenever $1 \leq r < m-1$.

Throughout one deals with the effect of the degenerateness on the numerical invariants and ideal theoretic properties of the gradient ideal of f and submaximal minors. We show that in the degenerated case the gradient ideal is never a minimal reduction and when $r = m-2$ is of linear type.

This talk is based on a joint work with L. Cunha, Z. Ramos and A. Simis.

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Positive Matching Decomposition of Hypergraphs

Marie Amalore Nambi
Indian Institute of Technology Hyderabad, India

In this talk, we give characterization of positive matching in terms of strong alternate closed walks for a r-uniform hypergraph $H = (V, E)$ such that $|e_i \cap e_j| \leq 1$ for all $e_i, e_j \in E$. For specific classes of a hypergraph, we classify the radical and complete intersection Lovász−Saks−Schrijver ideals. In the last part of the talk, we will present the conjectures of Gharakhloo and Welker (Hypergraph LSS-ideals and coordinate sections of symmetric tensors, *Comm. Algebra*, 2023) that the positive matching decomposition number (pmd) of a 3-uniform hypergraph is bounded from above by a polynomial of degree 2 in terms of the number of vertices.

This is a report on joint work with Neeraj Kumar.

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Finiteness of n-cluster Tilting Subcategories of the Module Categories

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Higher Auslander-Reiten theory was introduced and developed by Iyama. It deals with n-cluster tilting subcategories of module categories, where n is a fixed positive integer. Higher Auslander-Reiten theory has several connections to other areas, for example non-commutative algebraic geometry, combinatorics, categorification of cluster algebras, higher category theory and symplectic geometry. The question of finiteness of n-cluster tilting subcategories for $n \geq 2$, which is among the first that have been asked by Iyama, is still open. Up to now, all known n-cluster tilting subcategories with $n \geq 2$ are of finite type. In this talk after recalling the definition and some basic properties of n-cluster tilting subcategories we give several characterization of n-cluster tilting subcategories of finite type.

This talk is based on joint works with R. Diyanatnezhad, R. Ebrahimi and Z. Fazelpour.

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Combinatorics and Topology of Complexes of Multichains

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Let P be a poset with order relation $\leq$. For a number $r \geq 1$ we consider the set P_r of all r-multichains $\mathfrak{p} : p_1 \leq \cdots \leq p_r$ in P. In this talk, we define and study a class of simplicial complexes associated to the set P_r of r-element multichains. The simplicial complexes depend on a strictly monotone function from $[r]$ to $[2r]$. We show that there are exactly 2^r such functions that yield subdivisions of the order complex of P, of which 2^{r-1} are pairwise different. We also exhibit a large subclass for which our simplicial complexes are order complexes and homotopy equivalent to the order complex of P. It is also shown that all these subdivisions have the same combinatorial data, i.e., f-vector. We give an explicit description of the transformation matrices from the f- and h-vectors of Δ to the f- and h-vectors of these subdivisions when P is a poset of faces of Δ. We will also discuss two important subdivisions, namely Cheeger-Müller-Schrader's subdivision and the r-colored barycentric subdivision which fall in our class of r-multichain subdivisions.

This work is a partial collaboration with Prof. Volkma Welker.

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Fröberg's Theorem, Vertex Splittability and Higher Independence Complexes

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A celebrated theorem of Fröberg gives a complete combinatorial classification of quadratic square-free monomial ideals with a linear resolution. A generalization of this theorem to higher degree square-free monomial ideals is an active area of research. The existence of a linear resolution of such ideals often depends on the field over which the polynomial ring is defined. Hence, it is too much to expect that in the higher degree case a linear resolution can be identified purely using a combinatorial feature of an associated combinatorial structure. However, some classes of ideals having linear resolutions have been identified using combinatorial structures. In the present talk, we use the notion of r-independence to construct an r-uniform hypergraph from the given graph. We then show that when the underlying graph is co-chordal, the corresponding edge ideal is vertex splittable, a condition stronger than having a linear resolution. We use this result to explicitly compute graded Betti numbers for various graph classes. Finally, we give a different proof for the existence of a linear resolution using the topological notion of r-collapsibility.

This is a joint work with Priyavrat Deshpande, Anurag Singh and Adam Van Tuyl.

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Graded Components of Local Cohomology Modules Supported on $\mathfrak{C}$-monomial Ideals

Sudeshna Roy
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The structure of local cohomology modules is quite mysterious owing to their non-finite generation. Over the last three decades, researchers have extensively investigated if they behave like finitely-generated modules. Let A be a Dedekind domain of characteristic zero such that its localization at every maximal ideal has mixed characteristic with finite residue field. Let $R = A[X_1, \ldots, X_n]$ be a polynomial ring equipped with the standard multigrading. Let $I \subseteq R$ be an ideal which is generated by elements of the form aU, where $a \in A$ (possibly nonunit) and U is a monomial in X_i's. We call such an ideal as '$\mathfrak{C}$-monomial ideal'. Local cohomology modules supported on usual monomial ideals of a polynomial ring over a field gain a great deal of interest due to their connections with combinatorics and toric varieties. The objective of this talk is to discuss a structure theorem for the multigraded components of the local cohomology modules $H^i_I(R)$ for $i \geq 0$. We will further show that if A is a PID then each component can be written as a direct sum of its torsion part and torsion-free part. As an application, we will see that their Bass numbers are finite.

This is a joint work with Tony J. Puthenpurakal.

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Matroids and their Cycle Polytopes and Algebras

Sara Saeedi Madani
Amirkabir University of Technology and IPM, Iran

In this talk, we introduce a certain type of polytopes and toric algebras associated with matroids, called “cycle polytopes” and “cycle algebras”, respectively, and discuss several properties of them.

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Codes on Subgroups of Weighted Projective Tori

Mesut Şahin
Hacettepe University, Turkey

We talk about certain algebraic invariants relevant in studying (generalized) toric codes on subgroups of weighted projective tori inside an n-dimensional weighted projective space. As an illustration, we share the main parameters of toric codes on a weighted projective plane of the form $\mathbb{P}(1, 1, a)$ for an integer $a > 1$.

This is a joint work with Oğuz Yayla that is published in Designs, Codes and Cryptography. DOI: 10.1007/s10623-023-01337-y.

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Monomial Ideal Powers and Strong Persistence Property

Hero Saremi
Islamic Azad University, Sanandaj, Iran

Let $R = K[x_1, \ldots, x_n]$ be the polynomial ring in n variables over a field K and I be a monomial ideal of degree $d \leq 2$. We show that $(I^{k+1} : I) = I^k$ for all $k \geq 1$ and we disprove a motivation question of Carlini, Hà, Harbourne and Van Tuyl by providing of a counter-example. Also, by this counter-example, we give a negative answer to the question that the depth function of square-free monomial ideals are non-increasing.

This is a joint work with Amir Mafi.

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Geometric Goppa Codes

Saeed Tafazolian
University of Campinas, Brazil

Geometric Goppa codes constitute a specialized class of error-correcting codes that emerge from the intersection of algebraic geometry and coding theory, specifically utilizing tools from algebraic curves. Goppa codes have demonstrated remarkable performance in mitigating errors in various communication channels and storage systems. In this talk, we provide a succinct overview of algebraic geometric Goppa codes, elucidating their theoretical foundations and construction methodologies.

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Methods of Gluing Hypergraphs with Nice Topological and Combinatorial Properties

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Given an arbitrary hypergraph $\mathcal{H}$, we may glue to $\mathcal{H}$ a family of hypergraphs to get a new hypergraph $\mathcal{H}'$ having $\mathcal{H}$ as an induced subhypergraph. In this talk, we introduce three gluing techniques for which the topological and combinatorial properties (such as Cohen-Macaulayness, shellability, vertex-decomposability etc.) of the resulting hypergraph $\mathcal{H}'$ is under control in terms of the glued components. This enables us to construct broad classes of simplicial complexes containing a given simplicial complex as induced subcomplex satisfying nice topological and combinatorial properties.

This is a joint work with Mohammad Farrokhi D. G., and Alireza Shamsian.