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Boundedly Axiomatizable Theories

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Albert Visser and I recently published a paper (2024) on the incompleteness of boundedly axiomatizable theories. The main result of our paper shows that any consistent extension of a sequential theory that is axiomatizable by a set of axioms of bounded complexity has to be incomplete. In the theorem “bounded” refers to the *depth-of-quantifiers-alternation*, which in the context of arithmetical theories that are at least strong as $\text{I}\Delta_0 + \text{Exp}$, coincides with the usual Σ_n -complexity measure (in which Σ_0 corresponds to formulae all of whose quantifiers are bounded by a term). There is also a version of the theorem that applies to sufficiently strong set theories.

More recently, in joint work with Visser and Mateusz Lęłyk, we have obtained some complementary results that go in the opposite direction by showing that there are models of arithmetical theories weaker than $\text{I}\Delta_0 + \text{Exp}$ whose complete theory is axiomatizable by a collection of Σ_n -sentences (for fixed n). Our strongest result in this direction constructs a model of $\text{IOpen} + \text{Coll}$ (where Coll is the full scheme of collection) whose complete theory is axiomatizable by a single sentence plus a collection of Σ_1 -sentences.

In this talk I will present an exposition of the aforementioned developments.