Two days Seminar on
Mathematical Logic and its Applications,
May 28-29, 2025, School of Mathematics, IPM, Tehran

On Weakly O-Minimal Structures and Strong Cell Decomposition

Somayyeh Tari
Azarbaijan Shahid Madani University

A first-order expansion $\mathcal{M} = (M, <, ...)$ of a dense linear order without endpoints is said to be *weakly o-minimal* if every unary definable subset of M (with parameters from M) is a finite union of open convex sets and points. These structures generalize o-minimal structures by allowing definable sets to have more complex forms while preserving certain tameness properties. This talk explores some fundamental properties of weakly o-minimal structures. Specifically, we examine the notion of strong cell decomposition in this context and provide a criterion for when such a decomposition exists.

Formally, let $\mathcal{M} = (M, <, ...)$ be a first-order structure expanding a dense linear order without endpoints. Then $\mathcal{M}$ is called *weakly o-minimal* if for every formula $\varphi(x, \bar{a})$ with parameters $\bar{a} \in M$, the set

$$\varphi(M, \bar{a}) = \{x \in M : \mathcal{M} \models \varphi(x, \bar{a})\}$$

is a finite union of open convex sets and isolated points. This condition ensures that although the structure may not be o-minimal, its one-dimensional definable sets still have a relatively simple geometric description. Here we investigate the property of *strong cell decomposition* in weakly o-minimal structures. This property allows us to partition the domain into finitely many "cells" over which definable functions behave nicely. We aim to provide a criterion under which a weakly o-minimal structure admits strong cell decomposition.

In addition, we study *o-minimal traces*, which are a special class of weakly o-minimal structures. An o-minimal trace can be thought of as a structure obtained by naming certain subsets from an ambient o-minimal structure. We prove that such structures satisfy the strong cell decomposition property.