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Preprojective Algebras and Quivers with Potential (5 Lectures)

Martin Herschend
Nagoya University
Japan

To any finite acyclic quiver Q we can associate its preprojective algebra. If Q is Dynkin, then the preprojective algebra is finite dimensional, selfinjective and stably 2-Calabi-Yau (CY). If Q is non-Dynkin, then the preprojective algebra is infinite dimensional and 2-CY.

Now let A be a finite dimensional algebra of global dimension n. To A we can associate a similarly defined higher preprojective algebra. The results above have the following analogues in higher dimensional Auslander-Reiten theory. If A is n-representation finite (n-RF), then its preprojective algebra is finite dimensional, selfinjective and stably (n+1)-CY. If A is n-representation infinite (n-RI), then the preprojective algebra is infinite dimensional and (n+1)-CY.

Bernstein-Gelfand-Ponomarev reflection incarnated as Auslander-Platzeck-Reiten (APR) tilting also has a higher dimensional analogue. More precisely, if A has a simple projective non-injective module, then it gives rise to a so-called n-APR tilting module T. We call the endomorphism algebra of T an n-APR tilt of A. Up to a change of grading, n-APR tilting preserves higher preprojective algebras and the properties of being n-RF and n-RI, respectively.

For n = 2, the higher preprojective algebra of A is given by a quiver with potential (QP) and cut (a potential on a quiver is a linear combination of cycles and a cut is a set of arrows that intersects each cycle appearing in the potential exactly once). Combining this description of higher preprojective algebras with the results above we will demonstrate that 2-RF and 2-RI algebras are precisely given by respectively selfinjective and 3-Calabi-Yau QPs and cuts.

We will apply Derksen-Weyman-Zelevinsky QP mutation to construct new QPs from given ones both in the selfinjective and 3-CY case. Moreover, we will interpret 2-APR tilting as a change of cut called cut mutation. The usefulness of these methods will be demonstrated by constructing many examples of both 2-RF and 2-RI algebras.