Ampleness of Two-sided Tilting Complexes, Fano Algebras and AS-regular Algebras
(4 Lectures)

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In my lecture it is explained that there is a strong relationship between representation theory and (non)commutative algebraic geometry via derived categories.

A relationship between representation theory and algebraic geometry goes back to the results of Kronecker which gives a classification of irreducible representations of Kronecker quiver by using the projective line. Nowadays it is understand as a consequence of a derived equivalence between the category of representations Kronecker quiver and that of coherent sheaves on the projective line. This derived equivalence is a special case of Beilinson Theorem which gives a derived equivalence between the category of coherent sheaves on the n-dimensional projective space and that of modules of n-Beilinson algebras.

Path algebras of quiver and canonical algebras have been studied by Lenzing and his school by using geometric notions. They are just algebras of low global dimensions.

We introduce the notion ampleness of two-sided tilting complexes and explain that this gives a derived equivalence between the category of modules and noncommutative projective scheme (in the sense of Artin-Zhang). This gives a formalism to study algebras of higher global dimension by using geometric notions.

Using the notion of amplness, we gives the definitions of Fano algebra and its variant. Although the definition is given from the view point of noncommutative geometry, it is turned out that this class of algebra is of representation theoretical importance. (It will be explained in Herschend's talk and also Iyama's talk.)

We also show that n-representation infinite (n-RI) algebras (a class of Fano algebra) has a strong relationship with AS-regular algebras which is a main object of noncommutative projective geometry. In particular we see that AS-regular algebras is a good supply of n-RI algebras.