miércoles, 27 de enero de 2021 6:41 a. m.

Proposition

Let $\mathcal{C} = \{G_n : n \in \mathbb{N}\}$ be a class of finite graphs such that:

(a) Each graph G_n is r-regular (resp. d_n-regular)

(b) $\mathrm{girth}(G_n) \to \infty$

Then, every infinite ultraproduct M of graphs in $\mathcal{C}$ is a model of T_r (resp. a model of T_∞ if $d_n \to \infty$.

Theorem (G., Robles)

Such classes of finite graphs exists, therefore the theories T_r and T_∞ are both pseudofinite.

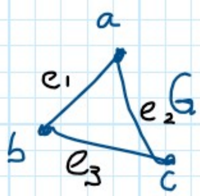
Lifting: $G \longrightarrow L[G]$

$G = (V, E)$

Vertices of $L[G]$: $V \times \{0,1\}^{E(G)}$ $\qquad (v, f: E(G) \to \{0,1\})$

$(v,f) \sim (w,g) \iff e = \{v,w\} \in E \;\&\; f\restriction_{\neq e} = g\restriction_{\neq e} \;\&\; f(e) \neq g(e)$

- If G is d-regular $\Rightarrow$ $L[G]$ d-regular
- If $\mathrm{girth}(G) = k$ $\Rightarrow$ $\mathrm{girth}(L[G]) = 2k$



$\Longrightarrow$

$(a, 000) - (b, 100) - (c, 101)$

$(c, 010) - (b, 011) - (a, 111)$

$G_n \longrightarrow$ r-regular, $\mathrm{girth}(G_n) \geq n$

$K^n_{r+1} \longrightarrow \longrightarrow \longrightarrow$ lifting n $\quad 2^k \geq n$

$\mathrm{girth} = 3 \to 6 \to 12 - - - \quad \geq n$

Theorem (G., Robles)

Let $\mathcal{C} = \{G_n : n \in \mathbb{N}\}$ be a class of finite graphs such that each graph G_n is d_n-regular and $d_n, \text{girth}(G_n) \to \infty$.

Let M be an infinite ultraproduct of graphs in $\mathcal{C}$ (a model of T_∞) and fix the non-standard integers $\alpha = |M|$ and $\beta = [d_n]_{\mathcal{U}}$. Then for every formula $\phi(\overline{x}, \overline{y})$ in the language of graphs there is a finite number of polynomials $p_1(X, Y), \dots, p_k(X, Y) \in \mathbb{Z}[X, Y]$ such that:

1. For every $\overline{a} \in M^{|\overline{y}|}$, $|\phi(M^{|\overline{x}|}, \overline{a})| = p_i(\alpha, \beta)$ for some $i \leq k$.
2. Moreover, there are formulas $\psi_1(\overline{y}), \dots, \psi_k(\overline{y})$ such that for every $\overline{a} \in M^{|\overline{y}|}$,
$$M \models \psi_i(\overline{a}) \Leftrightarrow |\phi(M^{|\overline{x}|}, \overline{a})| = p_i(\alpha, \beta).$$

This is enough to show that any class of graphs with the properties above is a multidimensional exact class.

1. Enough to check one-variable formulas

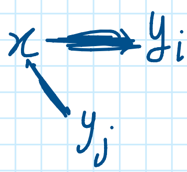
2. QE $\longrightarrow$ $\varphi(x, \bar{y}) = \bigvee_j \underbrace{\bigwedge_i D_{k_i}(x, y_i)^{n_i}}$

3. disjunctions can be avoided

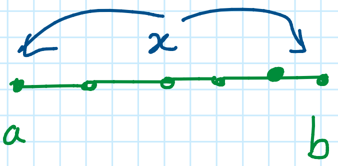
4. $\neg D_k(x,a) \wedge D_m(x,b) \Leftrightarrow \neg \left(\bigvee D_m(x,b) \wedge D_j(x,a) \right)$

$\bigwedge_{i=1}^{m} D_{k_i}(x, y_i)$ → "possible configurations of y_i's" (dist)

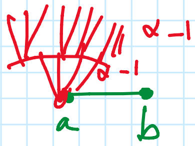
→ "possibilities for



$D_2(x,a) \wedge D_3(x,b)$



Polynomial	$\psi(\bar{y})$
1	$D_5(a,b)$
$(\alpha-1)^2$	$D_1(a,b)$



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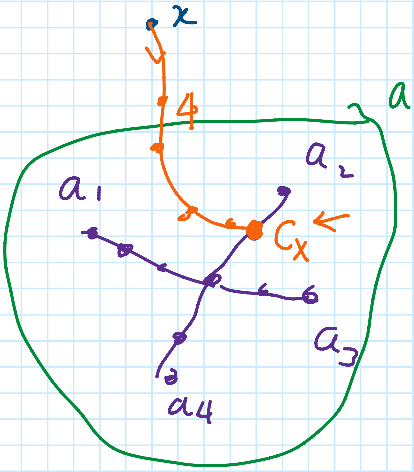
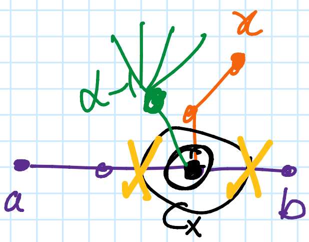


$\operatorname{acl}(a_1, \dots, a_4)$

$D_{k_1}(x,a_1) \wedge \dots \wedge D_{k_4}(x,a_4)$

$(\alpha-1)^3 \cdot$ (α-paths in acl)



$D_4(x,a) \wedge D_3(x,b)$

$(\alpha-1)(\alpha-2)$