

# دوره درسی

## مباحثی در ترکیبیات حسابی

### Topics in Arithmetic Combinatorics

عنوان درس:

مباحثی در ترکیبیات حسابی

ارائه دهنده:

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واحد:

۴ واحد درسی

دانشجویان هر سه مقطع کارشناسی، کارشناسی ارشد و دکتری

می‌توانند در این دوره شرکت کنند.

زمان: چهارشنبه‌ها

ساعت: ۹ الی ۱۰:۳۰ و ۱۱ الی ۱۲:۳۰

زمان برگزاری اولین جلسه:

چهارشنبه: ۹۸/۱۱/۳۰

سالن یک

متقاضیان برای اطلاعات بیشتر می‌توانند با ایمیل زیر در ارتباط باشند:

Email: A.Mohammadi@ipm.ir

آدرس: میدان نیاوران، پژوهشگاه دانش‌های بنیادی،

پژوهشکده ریاضیات، سالن ۱

# Topics in Arithmetic Combinatorics

## General information

- **Course Lecturer:** Dr Ali Mohammadi  
Email: A.Mohammadi@ipm.ir  
Office: Room 42
- **Time:** Wednesdays, 9am – 10:30am and 11am – 12:30pm
- **Venue:** Lecture hall 1, School of Mathematics, IPM
- **Consultation:** Wednesdays, 3:30pm–4:30pm or by appointments
- **Level:** This course is primarily intended for masters and doctorate students. Although, we will also accommodate enrolment of interested undergraduate students (e.g. by devising a separate assessment scheme), depending on the policies of their home universities.
- **Credits:** 4 units

## Course description

Arithmetic combinatorics deal with a wide range of problems, which typically arise out of the interplay between combinatorics, harmonic analysis and number theory. A major theme in this area is the sum-product phenomenon, which revolves around various questions that seek to establish, in a quantitative manner, the disjointedness of additive and multiplicative structure within subsets of a given ring.

In this course, we will study a number of topics surrounding the sum-product phenomenon over finite fields. This area has been developing quite rapidly over recent years and receives special attention, in part, due to its numerous applications to number theory as well as certain applied disciplines such as theoretical computer science, coding theory and cryptography. We will particularly focus on certain results from incidence geometry over finite fields and discuss their significance within the context of the sum-product phenomenon. We will also discuss the links between sum-product theorems and cancellation in various character sums.

# Syllabus

## 1. Basics of the theory of finite fields

- Basic properties; cardinality, characteristic, uniqueness, cyclicity of the multiplicative group of a finite field
- Construction of finite fields
- Additive and multiplicative characters

## 2. The polynomial method

- The number of directions determined by a point set
- Dvir's proof of the finite field Kakeya conjecture

## 3. Incidence theorems

- Murphy and Petridis' proof of Vinh's bound on point-line incidences
- Rudnev's bound on point-plane incidences
- Stevens and de Zeeuw's bound on point-line incidences
- The number collinear triples and quadruples determined by a point set
- The number of lines determined by a point set

## 4. The sum-product phenomenon

- Derivation of sum-product estimates from incidence theorems
- Balog-Szemerédi-Gowers theorems
- Balog-Wooley energy decomposition theorems

## 5. Estimates of character sums

- The Weil bound for additive and multiplicative character sums (only an overview)
- Estimates of various exponential sums based on techniques from arithmetic combinatorics and sum-product theorems
- Pólya-Vinogradov and Burgess bounds for multiplicative character sums

# Prerequisites

There are no serious prerequisites and the contents of this course will be accessible to anyone with adequate mathematical maturity. However, familiarity with the theory of finite fields, in particular its number theoretic aspects, will be helpful.

## Assessment

Assessment in this course will consist of a number of assignments, projects and weekly exercises. The exact details of the assessment will be announced during the first week.

## Important note

Students interested in enrolling in this course are strongly advised to check with their home universities regarding their policies in this matter. They will likely have to present IPM with a letter of approval soon after the start of the semester.

## Useful resources

- S. Ball and A. Blokhuis, Polynomial methods in finite geometry (available from <https://mat-web.upc.edu/people/simeon.michael.ball/braunsweig.pdf>).
- Z. Dvir, Incidence theorems and their applications, *Foundations and Trends in Theoretical Computer Science*, 6 (2012), 257–393.
- R. Lidl and H. Niederreiter, *Finite Fields*, Cambridge Univ. Press, Cambridge, 1997.
- A. Sheffer, Polynomial Methods and Incidence Theory (available from <http://faculty.baruch.cuny.edu/ASheffer/000book.pdf>).
- I. E. Shparlinski, Additive combinatorics over finite fields: new results and applications, *Finite Fields and Their Applications. Character Sums and Polynomials*, De Gruyter, Berlin, 2013.
- T. Tao and V. Vu, *Additive Combinatorics*, Cambridge Univ. Press, Cambridge, 2006.
- F. de Zeeuw, A course on sum-product bounds (available from [https://www.epfl.ch/labs/dcg/wp-content/uploads/2018/10/SumProductBounds\\_20170529.pdf](https://www.epfl.ch/labs/dcg/wp-content/uploads/2018/10/SumProductBounds_20170529.pdf)).