Course title**: Probability and Computing**

Fall 2023, IPM (Institute for Research in Fundamental Sciences), School of Mathematics, Niavaran Building; Time: Mondays (9 am – 12:30 pm); 1st session: 3rd of Mehr, 25/9/2023.

Instructor: Omid Etesami

This course is about the relation of probability and computing, more specifically randomized algorithms and probabilistic analysis of processes and data. The course is at the advanced undergraduate/beginning graduate level. The prerequisite for the course is mathematical maturity; one course in probability and one course in algorithms (or data structures) is recommended.

The basic core of the course consists of the following topics: 1. Events and axioms of probability; applications: verification of algebraic computations, naive Bayesian classifier, randomized min-cut algorithm. 2. Discrete random variables, expectation (linearity, Jensen's inequality), conditional expectation, binomial and geometric distributions. Examples: coupon collector, quicksort. 3. Moments and deviation: Markov and Chebyshev inequalities. Examples: variance of coupon collector, median-finding algorithm. 4. Chernoff/Hoeffding bound. Examples: parameter estimation, set balancing, and (time permitting) packet routing. 5. Balls and bins, birthday paradox, and Poisson approximation; hashing techniques: Bloom filters and symmetry breaking; random graphs and algorithm for Hamiltonian cycles.

We will also discuss the following topics which may be covered in other courses, but our focus is more algorithmic: 1. The probabilistic method: expectation argument (and algorithmic applications); derandomization using conditional expectations; sample and modify; second moment method; Lovasz local lemma and (time permitting) its algorithmic version. 2. Markov chains and random walks. Examples: satisfiability algorithms, s-t connectivity algorithm, gambler’s ruin, Parrondo’s paradox.

If time permits, we also discuss a selection of the following topics: 1. Continuous distributions, exponential distribution, the Poisson process; examples: Markovian queues, balls and bins with feedback. 2. Normal distribution: central limit theorem, multivariate normals, generating normal random variables, maximum likelihood point estimation, EM algorithm for mixture of Gaussians. 3. Entropy, randomness, information: approximation of binomial coefficients, compression and coding. 4. Monte-Carlo method, DNF counting, relation of counting and sampling, the Metropolis algorithm. 5. Coupling of Markov chains, mixing time, card shuffling. 6. Martingales, stopping times, applications of Azuma-Hoeffding's inequality. 7. PAC learning, agnostic learning, sample complexity, VC dimension, Rademacher complexity. 8. Pairwise independence, universal hash functions, data streams. 9. Power laws, preferential attachment. 10. The power of choice in balanced allocation, cuckoo hashing.

The textbook for the course is: Probability and Computing, Mitzenmacher and Upfal, Cambridge University Press, 2nd edition, 2017. It is important to use the second edition of the book.