دوره درسی

نظریه میدان کوانتومی توپولوژیک

عنوان درس:

Topological Quantum Field Theory

ارائه دهنده:

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واحد:

۴ واحد درسی

این درس برای دانشجویان دوره کارشناسی ارشد و دکتری می‌باشد

(در صورت تمایل، دانشجویان کارشناسی با در نظر گرفتن پیش‌فرض‌ها
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جلسه توجیهی و کلاس درس به صورت حضوری و آنلاین برگزار می‌شود.

در صورت تمایل به شرکت در کلاس، به آدرس
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Lectures on Topological Quantum Field Theory

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Abstract

As the title suggests, this course is dedicated to the study of Topological Quantum Field Theory (TQFT). We will begin with an introduction to quantum mechanics, gradually transitioning to functorial quantum field theory, and then explore Atiyah-Segal's axioms of TQFT within the framework of monoidal categories. From there, we will classify 1- and 2-dimensional TQFTs and study the role of Frobenius algebras. Next, we will introduce Dijkgraaf-Witten Gauge Theories, a simple class of concrete examples of TQFTs. The course will then dive into 3-dimensional TQFTs, exploring their connection to the classification of 3-manifolds. Finally, we will examine the work of Reshetikhin and Turaev, who relate modular tensor categories to TQFTs and their associated link invariants. Unfortunately, due to time constraints, we will not be able to cover topics such as Donaldson theory, Floer homology, or Chern-Simons theory. Perhaps these could be explored in a potential Part II of the course.

Who Is This Course For?

This course is open to both mathematics and physics graduate students, as well as strong undergraduates. No prior knowledge of physics is assumed; we will rigorously build the foundations from the ground up. The mathematical prerequisites are standard undergraduate coursework, but I wouldn't be overly concerned about them. A physics student with some mathematical maturity and enthusiasm should be able to quickly catch up on any topic as needed, and I will provide supplementary resources when necessary. Additionally, I am writing extensive lecture notes for the course, complete with numerous appendices that will help you catch up on any material as needed.

Tentative Syllabus

Below is a rough outline of the topics I plan to cover, listed in order. Naturally, I can provide fewer details about the specifics of the later topics at this stage. Things can change in the classroom, and I haven't yet finalized exactly what I want to say about the topics we'll discuss towards the end of the course.

Building Segal's Idea of a QFT from Scratch

1. Classical Field Theory
   (a) Fiber Bundles
   (b) Classical Fields are Sections of Fiber Bundles
   (c) Action Principle and Locality Principle
   (d) Euler-Lagrange Equation
   (e) Example: Classical Mechanics
   (f) (Time Permitting) Klein-Gordon Field Theory
2. The Principles of Quantum Mechanics
   (a) Quantum States and Hilbert Spaces
   (b) Observable and Self-Adjoint Operators
   (c) Wavefunction Collapse
   (d) Probabilistic Nature
   (e) Transition Probability and Unitaries
   (f) Schrödinger's Equation
3. Dirac's Formalism
   (a) Schwartz Space $\mathcal{S}$
   (b) Tempered Distributions $\mathcal{S}'$
   (c) Gelfand Triple $\mathcal{S} \subset L^2 \subset \mathcal{S}'$
   (d) Schwartz Kernel Theorem
   (e) Dirac's Bra-Ket Formalism
4. Feynman's Path Integral
   (a) Feynman's Thought Experiment
   (b) Free Propagators
   (c) Heuristic Formulation of Feynman's Path Integral
   (d) Wick Rotation
   (e) Feynman-Kac Formulation
5. Functorial Quantum Field Theory
   (a) The Mythology of Path Integrals in Field Theory
   (b) Segal's Idea of Functorial Quantum Field Theory
   (c) Observable in Funtorial QFT
   (d) Example: Quantum Mechanics

Definition and Examples of TQFTs

1. Cobordisms: The Category of Spacetimes
   (a) Orientation
   (b) Idea of Obstruction Theory and Steifel-Whitney Class
   (c) Unoriented Cobordisms and Steifel-Whitney Numbers
   (d) Oriented Cobordisms
   (e) Introduction to Category Theory
   (f) Catgeory of Spacetimes
2. Dagger Compact Categories
   (a) Monoidal Categories
   (b) Symmetric Monoidal Categories
   (c) Compact Closed Categories
   (d) Dagger Compact Categories
   (e) Examples
3. Atiyah-Segal's Axioms & 1D TQFTs
   (a) Atiyah's Monoidal Functor $Z : \mathrm{Cob}_n \to \mathrm{Vect}_{\mathbb{C}}$
   (b) Classification of 1D TQFTs
4. Frobenius Algebras & 2D TQFTs
5. Dijkgraaf-Witten Gauge Theories

3D TQFTs and Low Dimensional Topology

1. Tangles, Links, and Basic Invariants
2. 3-Manifolds and Dehn Surgery
3. Braid Structure and Ribbon Categories
4. Modular Tensor Categories
5. Reshetikhin-Turaev Link Invariants